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MINIMALISM of INFORMATION

PHYSICAL INFORMATION AND APPLICATIONS

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Foreword

This book is an immediate extension of Physical Information [2]. The idea of basing information theory on objective coincidence, which in the previous book focused on the conservation of information, extends further to the principle minimalism discussed here, but also to the uniqueness not mentioned. There is too much mathematics in these theses that they cannot be easily digested, so it makes sense to try to explain "minimalism" even here in the foreword of the debate.

When we have too little information then we are uninformed, but if we are over informed then we are misinformed. It is call the principle of information, and we somehow recognize it in this form. If we add that "freedom" is a type of information then it also may be too little or too much. It is a factor of "ability" and "restriction" over the same opportunities that lie around us.

Opportunities will better express the more able ones, especially in more complex situations, the difficulties of which we measure "constraints". Consequently, total freedom is the sum of products of value pairs; it is a function that has a local extreme, an optimum. Moreover, for example in macroscopic looking, with the subject's over-ability, the intensity of the constraint becomes null and void along with the corresponding freedom. The opposite is also true, with great limitations, too much objective difficulty, abilities are fading, so the result is again not-freedom.

Can we intuitively feel that an absolute (very many) capable being would be absolutely unfree (very bound) as well as being absolutely limited by difficulties, I do not know. That is why I try to treat this topic not only philosophically but also formally mathematically. The second reason, of course, is that history is riddled with café theories and ideas that have not undergone rigorous mathematical checks, or are not experimentally verified later, most often neither of these two.

The purpose of the second part of this book, mathematical formalism, is also educational. At the time I was writing this, there were worrisome few who had elementary knowledge of quantum mechanics, even when they were from professions bordering on that field of physics. That part is therefore a compromise of the textbooks and (hypo) theses I was dealing with in those days, and then it happened again that there were other pieces to be thrown out for one more book.

R. Vuković, August 2019.

Sadržaj

Glava 1

Popular stories

Introduction

When, for the sake of (my) anonymity, we once agreed to call this a "science fiction based on mathematics", we can additionally say that it is an attempt to explain the world through information. Privately speaking, I have no serious doubts that in future times a theory of information similar to this will prevail in the sciences, natural and social, as much as I fear that such might become a dogma. It is absurd for anti-dogma science to become dogma, but it regularly threatens to strange human nature.

Although I do not burn out to popularize the ideas that follow, you will notice that the questions I was asked about are cautionary answered, or if you consider knowingly, that the texts were carefully selected. It's hard to be a "truth and only truth" ideologist in the new terrains, because truths like to hide. That is why we see masters in lying, in associations, or in bold decisions as lightly and "unjustly" more successful than others, as a threat to the "true" lovers of truth by their often offensive attitude towards their sanctity. I justify myself by this for my sometimes deliberate attempts and failures.

Well, in the columns that follow, the "truths" were generated independently or in response to questions that I had archived months before the date of publication in the portal Izvor[1], and almost this whole book was written a year before proofreading. I do not believe in coming up with major mathematical or scientific truths through public debates, so I avoided discussions. That is why my first agreement with the editor came from the belief that few people read such articles and that those who would, not want to argue with them. This portal is presented to me as a site of political texts and opposition, which is "as is known" only for the bunkum of "dogs barking as caravans pass" (proverb). Unfortunately, not everything turned out quite like that.

However, the basic idea is not completed here. Just as Darwin's evolution is a principle less than something exactly provable, so is the principle of probability, that is, information. More likely events are more frequent and less informative. When something is certain, we know that it will happen and when it finally happened, we do not consider it as some big news. In addition, information is an action, a product of energy and time, so the difference of potential energies in the force field is precisely caused by the principled minimalism of information. Known or unknown force fields are always areas of scalar information, and the point of this on physical forces is left for my next book.

[1] `http://izvor.ba/`

1.1 Frequency

How can it be that events are unrepeatable and so limitless in this supposedly Platonic *world of ideas* in which there is no set of all sets, theories of all theories, the best criterion, and everywhere we see material, finite and periodic phenomena?

That should be the first question for a theory that seeks to explain reality and especially one that would generalize the classic *information theory*. Another question would be about its development – what could be its significance?

Presenting everything with information, which is only one particularly measured *amount of data*, is a reduction of *perceptions* with that particular abstractness that gives it the breadth inherent in mathematics. Hence, the following, both simple and fast, as well as unexpectedly deep conclusions, are possible.

Information is (locally) unique because we (as well as particles) exchange messages because we do not have everything we need. For the same reason, the cause of communication is *unpredictability*, the existence of objective uncertainty, so contrary to the common belief that we know the past and hope for the future, we see better the consequences than the causes. Not only are our perceptions blurred with indeterminacy, but this world is so structured.

The glass on the table is exactly where it is, because it is its most likely position, and because of that it will be there in the next moment, unless some force is exerted on the glass and move it. *Force* changes the probabilities, changes the energy (force work on the path) of an object, changes our perceptions of objects (force work on a path for a given time) and testifies that similar assumptions lead to similar immediate consequences (not exactly equal). Thus we understand that we look at physical bodies through information, that this "world of information" is equivalent to material, that it is equally complete and contradictory.

The *conservation law* of the quantity of matter is transferred to the conservation of information, and the principle of the least action of matter to the equivalent *principle of minimalism* of information. From the first follows its finite divisibility (an infinite set may be equivalent to its real part as opposed to a finite one). It follows from the second that any information can be attributed to some action.

Because information is always finally divisible, its limited multiplicity is always finite, so the number of all combinations of that multiplicity is finite. Sooner or later, it repeats itself, and similarly leads to immediately similar, so all material phenomena are *periodic*. Periodicity itself, however, is a type of information, at least as information about much information. That is the answer to the first question above. The second will be clearer through the applications.

For example, consider the economics of the model market *Cournot*[2]. The goods on the market that he was talking about may be fuel, cloth, milk; it is all the same, as long as it is of the same type and until its unit price generally falls with the increase in supply. He used to write completely unknown works about it, to contemporaries too overweight which were only a century later recognized and included in the then discovered game theory. Explaining Cournot without formulas is considered impossible but worth a try.

The product of price and quantity sold is total *revenue*, and all other costs are *expense*. The difference between income and expenses is *profit*. As the volume saturates the market, the (unit) *price* decreases and the growth of total revenue slows down, and the expense grows steadily, so profit has some *optimum* in relation to quantity. After the optimum, with

[2] Antoine Cournot (1801-1877), French mathematician.

the increase in production, the business goes into minus, the loss of the company grows.

When a company is on the market itself, we say it has *monopoly* (goods given). When two companies compete for the same goods, it is called *duopoly*, and when there are more companies it is *oligopoly*. *Competition* (duopoly or oligopoly) produces some additional price reductions in the battle of firms' placement, so it happens that a monopoly firm in the market would achieve the optimum with less goods and a higher price than the optimum of competing firms. In other words, monopoly is good for the manufacturer, competition for society.

In a given market, each manufacturer has its own optimum production depending on the supply of the others. When that supply is too small for market capacity, the manufacturer can increase production and revenue, and if that supply is too large, it will go into losses. That optimum of commodity quantity is the *equilibrium*, or (often moving) center of oscillation of competitors, with the variable smaller and larger supply over time.

Some in this "oscillation game" fall away, others appear. The *equilibrium* state of the producer, relative to the market and the commodity, is called Cournot-Nash Equilibrium. What are further interesting are the aforementioned periodic phenomena, as the more interesting as its applications here are novelty.

Each *oscillation* is bound to some information, and any information is to some oscillation. In addition, we note that "information about crowd of information" can also be elementary. Then, the information is an action (change of energy for a given time), which means that elementary information and their periods can always be redefined with "energy", so that these products are constant, quantised.

In other words, the supposed *energy* is proportional to the *frequency* change (inversely proportional to the period), as is the energy of a wave-particle of light (photons) where the electric and magnetic fields alternate by inducing each other by motion.

This observation becomes practical when, instead of the complex relationships of competing firms, they switch to treating their circling frequencies around the equilibrium. It's an analogy with energies in physics that we know can simply be summed up, so the interference seems to go beyond *physics*. Just as *mathematics* helps physics with its models, the physics can now help mathematics, and both to *economics*, biology, sociology. Finally, the question is how different are these areas?

`http://izvor.ba/`
July 10, 2019.

1.2 Darwin's evolution

Darwin's[3] evolution supports the theory of physical information because it acknowledges choices. So an anonymous colleague notes, and he asks me: Is there something in this new theory that would support Darwin's evolution? This question leads to a difficult topic and an important positive answer to the very basics of the theory of physical information, and I hope to know how to elaborate it interestingly.

I actually consider physical information theory a mathematical theory, but I model it according to the principles of conservation, minimalism, and uniqueness of information that (mutually unambiguously) maps it to the material world. Only the first two are sufficient

[3]Charles Darwin (1809 - 1882), English naturalist, geologist and biologist.

for the definition of life, but only the first of these principles is more or less known to science, because it is still a topic of inquiry.

I will remind you that the law of conservation has until recently been largely suspected, so even the famous *Hawking*[4] once claimed that *black holes* "eat information" because the powerful gravity created by these celestial bodies does not allow anything to come out of them, not even light.

Many of scientific papers then "confirmed" the idea of non-conservation of information, for example, by referring to a piece of paper with text we burn and destroy the information of the text forever. Only recently did the famous physicist *Susskind*[5] Explain why black holes do not consume information, and Hawking himself accepted the explanation.

In short, as the body falls towards the *event horizon* of the black hole, its relative time slows to zero and its radial (toward the center of gravity) length is shortened, and the process itself takes infinitely long. From the outside, the body tends to become a two-*dimensional* image on a sphere around a black hole, never leaving the outside world. Its 2-D information, the very essence of its matter, in all its initial quantity constantly stays with us. That would be cosmological evidence of the conservation.

In quantum mechanics, the *quantum states* (sets of particles) are represented by vectors of so-called *Hilbert space*, and the changes (physical evolution) of these states by *unitary operators*. The point is that the unitary operators are linear, unitary, and reversible, which means that changes in quantum states keep many quantities constant, including information. That should be enough, because *quantum mechanics* is the most accurate branch of physics today, and therefore the most accurate theory in the natural sciences in general.

However, I said that "mathematical theory of physical information" is not really a theory of physics, so proving it by physical experiments does not apply. That is why I emphasize that quantum mechanics is a representation of Hilbert's algebra of abstract vector spaces, and the aforementioned unitary operators, for the sake of reversibility, provide the symmetry of the quantum states (before and after operation) and then, according to *Noether's theorem*[6], if we have symmetry then we have an appropriate law of conservation. And that is that.

Conservation of information is intuitively visible too. Namely, if information could come from nothing and disappear into nothing, then we could not believe in physical experiments. The past would not be scientifically researchable; it would change irrelevantly with the emergence and disappearance of information. Memory would be pointless and the lose the meaning of communications in general.

So much about the conservation of information. *Minimalism of information* follows, for example, from the view that the most likely events are most often realized and such are the least informative. Namely, when we know that something is going to happen and it happens, then it is not some news. Minimalism in physics is seen in the spontaneous growth of the entropy of the thermodynamic system, when gas molecules are arranged with one another (equalizing each other distances) reducing communication to the outside, which from the outside is viewed as an increase in disorder. Faster spreading of half-truths then truths by social networks, or easier coding than decoding, are also the consequences of skimping the nature of information emissions.

The same minimalism is a kind of frugality that accumulates information. It can be

[4]Stephen Hawking (1942-2018), English physicist.
[5]Leonard Susskind (1940-), American physicist.
[6]see Physical information [2], Emmy Noether.

proved (my theorems in [2]) that "physical information" is increasingly in comparison with "technical" (classical, Shannon's), the more complex the system is. Therefore, *living being* may have excess information due to its physical complexity.

The bodies of the excess tend to get rid of their information (the same principle of minimalism), but this does not go easy, because all the surrounding substance is filled. Living beings thus spontaneously can die on installments through interactions, conveying information to non-living substances in auspicious events, or resolving excess by organizing. With this spontaneity, people age or give up personal freedom (information) for the benefit of social organization. We can literally say that life and the legal system are killing us. Life wanted to be created as it did not want to be created, and this dualism is the specificity of information, it's the principle of minimalism. I mention again, as yet unknown in science.

Just this spontaneous accumulation of information into less complex life forms, and then the transfer of information from less complex individuals to a complex collective, which by evolution may (but may not) become a new more complex living being, is an important link that has been lacking in Darwin's evolution. Conservation of information and the principle of minimalism are the driving forces of the evolution of life, because only a mere random selection would tend to clutter and not to highly organized life forms.

`http://izvor.ba/`
July 19, 2019.

1.3 Information of perception I

Information of perception is a special type of amount of options an individual can experience. It is a measure of its developmental ability – it is an abbreviated answer to questions about the meaning of the "strange" title of my eponymous book [10]. Viewed in the abstract, this formula may say more about our relationship with the world or the communication of substance in general, than of each of our particular representations. I'll try to explain it.

It is enough to have at least some options and we can already define *intelligence* as the ability to choose. Unlike the results of the IQ test, the same term further encompasses both hereditary and acquired ability to manipulate opportunities. It still allows for its plasticity, adaptability, but also application to unconscious skills. Perceptions include observations from the unconscious, so the new definition of intelligence is consistent in this regard.

Individual ability to choose is limited and finite. Let's call its boundaries *hierarchy*. For the constraints imposed on the individual by the environment, we can use other terms, but in any case we mean that they come from legislators, social norms, instincts, natural laws. Also, within each of the above or similar prohibitions, the freedom of one person is limited by the corresponding freedom of others. Any ability to handle the outcomes of one affects the other, because the outcomes are conserved. As uncertainties, outcomes constitute (physical) information and are subject to the law of conservation.

Intelligence is directly proportional to *liberty* and inversely to hierarchy. The first says that intelligence (on average) seeks its comfort in greater freedoms (quantities of options). We recognize these in living beings as avoiding cramped states of individuals or evolving species into new possibilities in the process of adaptation to the environment for better use of resources. In the inanimate, we see the same in avoiding unnecessary emissions of information (from uncertainty and in general), in the principle of insincerity, its principled minimalism. Hence the information of perception, that is, the freedom that the individual can experience, is equal to the product of its intelligence and hierarchy.

Here is an interesting Frank *Ramsey* theorem that states that there is no zero hierarchy. To paraphrase, no matter how randomly distributed the clouds in the sky, sooner or later some preset shape will appear, or however randomly assigned letters and words to the text, there is always a chance to get a before-given sentence. Otherwise, this theorem was discovered in the early 20th century with the development of graph theory. With the supposed objectivity of possibility, it now establishes the absence of zero freedom and intelligence.

Objectivity of choice implies diversity, and independence of some phenomena. That is why the aforementioned formula, information of perception, must be more complex. At the very least, it contains individual freedoms as the product of an appropriate pair of amounts of intelligence and hierarchy. These products then participate in the total perception information as a sum, due to the laws of conservation. We do not "calibrate" these "amounts" for the sake of greater generality, but we can use examples.

Caesar's ability when crossing the Rubicon in 49 B.C. was opposed to the law of the Roman Senate, which forbade such a crossing. The greater the power of prohibition and the greater the ability of Caesar, the greater their product and the greater the *vitality* of Caesar (in that case), that is, the greater it's corresponding component (item) of the perception information.

A similar example is the strength of the game of competitors and opponents in some competition. Then again we can say that the "liberty" of the game grows in proportion to the skill of the player and the resistance of the opponent, the wealth of choices, so it is correct to consider simpler games those that seek less skill. Summing up all the individual liberties into the total information of perception, we find the form of a scalar product, here the vectors of intelligence and hierarchy. This is another difference between the new and the classic definition of intelligence – this one is a vector (string) and the old one is a scalar (number).

Because (total) information perceptions of individuals are more or less limited but intelligence is plastic, in a constant hierarchy, if we reduce our options in one domain they will try to expand in others, like fresh sausage squeezed at one extremity and explodes on the other. In support of this will be a manager who leads a rigid and boring life in less important matters and shows increased success and creativity in others. Accordingly, monogamy and its patriarchal mechanisms have been more successful in creating a civilization precisely because of the nature of its restrictiveness.

Finally, let us consider another purely computational property of the aforementioned scalar series multiplication in a non-computational manner. Let's compare the components of a string (vector) of information in ascending order, and the hierarchies in descending order, so we multiply pairs and sum all the individual products. By multiplying this, smaller with larger and larger with smaller, we get less information of perception than any other arrangement, multiplication and addition. We will have maximum value when we multiply the larger components of one string with the larger of the other and the smaller with the smaller. This also has its practical interpretation.

The minimum information of perception (liberty) is held by dead things, the subjects of the study of physics. All of them are subject to the known *principle of least action*, the smallest possible energy consumption to get from one point to another, or the minimal amount of time consumed in the circumstances. These would be "living creatures" released as a piece of wood down the water. Such laggards are less opposed to the greater obstacles, in contrast to the defiant Caesar who did just the opposite.

That is why it makes sense to call perception information vitality. The general denial

of options that comes from the fear of uncertainty, such as the need for security, stability, or efficiency, now becomes a stifle of development, and in conditions of otherwise limited perceptions, in the long run it leads to a decline in general intelligence. We see it all from that one abstract formula. If she is correct.

`http://izvor.ba/`
July 26, 2019.

1.4 Information of perception II

You call *freedom* (liberty) the sum of products of the corresponding *capabilities* and *restrictions*, and also *amount of options* alluding to *technical information*, amount of uncertainty – a detail from my interview with a colleague – but what is the connection between the two definitions? Here's an explanation.

The technical definition of information was discovered in 1928 by *Hartley*[7] working at Bell Telephone Company. He noted that the *logarithm* of the number of equally-probable data is a better measure of the amount of uncertainty than any other scale, and especially of the immediate number of possibilities.

There are two outcomes in a coin toss, six in a die throw, and 12 in a throw of them both, therefore not the sum but the product of the number of options, and the logarithm the product equals the sum of the logarithms. Logarithm is the only such additive function, so Hartley's definition was a jack pot. The telephone company could begin to "count" the data flow as correctly as the water mains the water consumption, or the electricity company consumed electricity.

From the additivity of logarithms it follows that the logarithm of the unit is zero, so there is so much information of certainty. A fair coin gives two equal outcomes, each with a probability of half to give the sum of both a unit and a certainty. A surefire event in a fair-dice toss is one of six options, so each has a sixth chance.

In ten some equal opportunities each has a probability of a tenth, a number reciprocal of the number ten, and a product of ten and a reciprocal of ten (six and sixths, two and half) is one, so the logarithm of the tithe is equal to minus the logarithm of ten. Hence, Hartley's information is minus the logarithm of probability. Changing the log base only changes the units of information.

The inverse of the logarithmic function is exponential (of the same base). This means that they cancel each other out so that the logarithm of the *exponent* number is equal to the given number. Therefore, if the given number is the said product, of the capabilities and limitations, the summand of liberty, then its exponent is the corresponding "number of options" for Hartley's information. The reciprocal of the number of options is some mean value of the probability of the option, and its negative logarithm is again the same information. That's the thing!

An individual product of the corresponding "ability" and "constraint" defines the component (item) of "freedom", its exponent defines the "number of options", and the logarithm of that number is again the same starting "freedom". However, Hartley's information can be recognized. Everything becomes crystal clear when the formulas are put on paper, but something is understood here as well.

The famous *Shannon*[8] definition of information came twenty years after Hartley's, from

[7] Ralph Hartley (1888-1970), American engineer.
[8] Claude Shannon (1916-2001), American mathematician.

the same company. Simply put, Shannon observed the oddly probable outcomes, divided them into groups of equally probable, and assigned each Hartley logarithm to each group, and then took the mean of the logarithms by probability distribution. In technology, this mean has found great use and has caused the explosion of computer development. But Shannon's information is not physical, since it does not follow the law of conservation as Hartley's[9].

That's why I'm using a refinement of the Shannon definition that is supported by the law of conservation, which I call *physical information*. It is larger than the Shannon as the system is more complex, which is shown in accordance with the *principle of minimalism*, then with the accumulation of information, and finally with the definition of "living being".

On the other hand, along with the development of classical information theory, quantum mechanics was also emerging. An important discovery from 1927, which we just learn to bring them together, was *Heisenberg's*[10] *uncertainty relations*. In their original form, they say that the products of the uncertainties of measuring the momentum and position of a particle, like as energy and time, are never less than a constant order of *Planck's*[11].

This product is *action*, here "freedom" or Hartley information, to say the space of a particle is its "ability" and its momentum is its "restriction". If we stick to the mathematical form, it is clear that it goes into equally non-contradictory views of the said "freedom", i.e. physical information, such as physical actions.

That the law of conservation is valid for physical action follows from its quantization and constancy. Since the action is information, its exponent is probability. Unlike the information just defined, in quantum mechanics it is known that said exponents represent probabilities of quantum states, and both the vectors in Hilbert space, i.e. sets of particles in physics. Things get further complicated because the components of the quantum state vector do not go with real ones but with *complex numbers*, but that makes sense now.

Only the corresponding expressions of these complex vectors when they are a real numbers become physically measurable quantities, the so-called *observable*, now in accordance with the theorem on the discreteness (finite divisibility) of each property of information. In addition, the degrees and logarithms of complex numbers are periodic functions, which is consistent with (also mine) claim that all information is periodic.

It is not a novelty that we choose solely observable, physical quantities that can be measured, for the coordinates of quantum states. So that the projections of vectors on them are the results of measurements expressed by probabilities. They are the components of *superposition*, as we called the alternatives of quantum measurement of a given state, and now we just add that the probabilities of measurement come from information or uncertainty that quantum states have. These are the "liberty" at the beginning of this text, which we might also call "information of perception".

Misunderstandings of the new with centuries-old interpretations of these terms come from their earlier daily use, from inconsistencies, often inaccurate and contradictory to their prior apprehension, not from the mathematics behind it all. This correction of meaning is one of the aspects of the progress of science.

`http://izvor.ba/`
August 2, 2019.

[9] It is discussed in detail in the book Physical Information [2].
[10] Werner Heisenberg (1901-1976), German theoretical physicist.
[11] Max Planck (1858-1947), German theoretical physicist.

1.5 Maxwell's demon

The Maxwell Demon is the thought experiment of the famous *Maxwell*[12] of the Second Law of *Thermodynamics* of 1867: thermal energy (heat) spontaneously moves from the body higher to the adjacent body at a lower temperature; never the other way. This was in the century of discovering molecules and learning that a higher speed of movement means higher *heat* and *temperature* of the body they make.

Well, we imagine a demon as a man controlling the partition between two parts of a room, two rooms with some advanced, unknown to us, physics and technology. It passes fast and only fast molecules from the first room to the second, and slow and only slow from the second to the first.

If the heat and temperature of the rooms were the same in the initial state, the first room would become colder over time and the second warmer. So the demon would send the heat from the colder room to the warmer, despite the second law of thermodynamics. The thermal difference between the rooms could give a new *useful work* and the demon would be a candidate for *perpetual mobile*, for an unlimited energy producer.

Thermodynamics was founded by the French military physicist Sadi *Carnot*[13] analyzing in 1824 an imagined heat engine of maximum efficiency. He came to the conclusion that the work produced (energy) cannot be greater than the invested and that only in the "ideal" case can the two be equal, and this is not the case in practice. It must be that energy is leaking through the walls of the pan, he noted.

These circular thermodynamic processes were further particularly carefully analyzed by the mathematician *Clausius*[14] in 1854 to publish his famous work on the theory of heat in which he established mentioned the second law of thermodynamics, holding for the first that energy can change its forms but not the total quantity.

Interestingly is his abbreviation, the quotient of heat and temperature, which he called *entropy* (Gr. inward orientation), until then completely unknown, and which he used extensively in formulas. At that substitution of variables, entropy, Clausus never gave any physical significance, but noticed that its value remains constant in the ideal Carnot cycle and increases in the real one. Later, entropy was referred to as the amount of disorder created by the titration of molecules (*Boltzmann*[15], 1877), and also information (*Shannon*[16], 1948).

Let's go back to the Maxwell demon. *Landauer*[17] observed in 1960 that thermodynamically reversible processes did not increase entropy, but at the cost of not allowing molecular information to be deleted. *Bennett*[18] further showed (1982) that the demon sooner or later must run out of storage space and begin deleting it, which will make this process irreversible causing an increase in entropy. He has proven that by losing information, the circular system loses energy, becomes irreversible and entropy grows!

Additional, recent calculations (Benet, 1987 – *Sagawa*[19], 2012) showed that the demon would produce more entropy by dealing with molecules than eliminating it by separating

[12] James Clerk Maxwell (1831-1879), Scottish mathematician and physicist.
[13] Sadi Carnot (1796-1832), French engineer and physicist.
[14] Rudolf Clausius (1822 - 1888), German physicist and mathematician.
[15] Ludwig Boltzmann (1844-1906), Austrian physicist and philosopher.
[16] Claude Shannon (1916-2001), American mathematician.
[17] Rolf Landauer (1927-1999), German-American physicist.
[18] Charles Bennett (1943-), American physicist.
[19] Sagawa, Takahiro (2012). Thermodynamics of Information Processing in Small Systems. Springer Science and Business Media. pp. 9–14. ISBN 978-4431541677.

them in rooms. In other words, more energy is required to evaluate and selectively leak molecules than would be obtained by the temperature difference of the rooms. This entire means that the demon is not possible, that the Clauses' laws are true and that there is a scientific future ahead of thermodynamics.

Note that the increase in entropy, followed by the loss of energy, can now be reduced to *principle of information* (minimalism): the nature is stingy with information. Heat spontaneously moves from the body of higher into the body at a lower temperature, because this reduces the emission of information. The uniform arrangement of the room molecules achieves the internal order at the expense of the loss of external communication that we perceive as a mess, and it is such a process of substance against which there is no cure.

The increase in entropy reduces the emission of information in the following ways. The number of combinations of uniform arrangement of molecules is much greater than the way they are piled, so in the case that all distributions are equally likely, even combinations are much more probable. That is the explanation Boltzmann explored. From the aspiration to the more likely it follows that nature tends to be less informative.

Secondly, we know that by randomly selecting words from a dictionary we get text in a mess, uninformative, while counting the words of a conversation would find statistically significantly different frequencies (number of occurrences) of individual words. If these words were arranged horizontally (along the abscissa) in decreasing height of frequency, in the first case (impersonal text) we would get an approximately horizontal line of heights, in the second case (meaningful text) we would have a descending curve.

Listening to and counting the signals (say, acoustic) emitted by animals (dolphins), according to the shape of the curve, if it is descending, we would know that they are talking, although then we could not recognize the meaning at all. It is similar to the uniform arrangement of the air molecules in the room which is externally looking impersonal to us, uninformative. It is a condition that is obtained by increasing entropy when we say that the mess is growing, ignoring the increase in internal order. Then, in fact, the internal order grows, and the external communication decreases.

The search for the cause of the concept (source) of energy is scarcely driven by a point on the principle of (minimalism) information, otherwise universal to all physical phenomena. This principle is yet to be seen, so in modern physics we still treat information differently (mechanical, thermodynamic, or electrical). The assumption is that, along with conservation laws, it hides even deeper connections between energy and information.

`http://izvor.ba/`
August 8, 2019.

1.6 Compton effect

It's amazing how much the *information of perception* blends into everything – a fellow computer scientist, otherwise an electrical engineer, asks me doubtfully. I answer: like computers that are no longer just a matter of algebra logic and electrical switches.

We are not surprised by the physics of atoms when they use them for their research, so soon will not even informatics be when it goes into the natural sciences from the principled theoretical studies. It would be a miracle that the overlapping of these theories will never happen and that we do not already have some precursor to such encounters.

One of the newly discovered connections between physics and informatics is from the well-researched scattering of *photons* (waves-particles of light) in collisions with *electrons*.

This is already learned in high school as the Compton Effect. The occurrence of the loss of a fraction of the energy in the collision, the photon with increase wavelength and that was predicted and described by the American physicist Arthur *Compton*[20] in 1922 considering the duality of wave and particle properties of electromagnetic radiation for which he received the 1927 Nobel Prize in Physics. I will use it to draw attention to the fine methods of information theory.

The formula, which Compton once discovered, predicts that the x-ray wavelength of 0.02 nano meters, if it hits an electron at rest and bounces at a 30-degree angle, increases its wavelength by about 16 percent, and then the electron flies off at an angle 73.5 degrees at a speed of about six percent of the speed of light.

When the same photon bounces at an angle of 45 degrees, its increase in wavelength is more than 35 percent, and the electron bounces at an angle of 65 degrees at a speed near 10 percent of the speed of light. Results similar to these are measurable and many have been carefully vetted and validated to date.

When, after colliding with an electron, the photon veers off its direction of motion, its wave is extended, its smearing increases. This is a decrease in the determination of the photon position and a decrease in the probability density of finding a photon at a given place, and thus an increase in the corresponding information – we note additionally.

Since nature more often realizes more probable events, which means less informative, consequently the photon would rather continue its previous straight line motion in accordance with the principle (minimalism) of information. From the same, forces and collisions cause changes in relative probabilities and then directions of motion.

In general, bodies move *inertial* (uniformly straight line) because they see the transition to a non-inertial or other inertial system of motion as a transition to states of higher emission of information, in states less likely. The other, relative states of increased communication for them are states of less *entropy*, because entropy grows by reducing the emission of information outward, and because of the desire for greater entropy of the body they remain in states of rest or uniform straight line motion until they are influenced by another body or force.

The disturbance of the maximum entropy of the system in uniform motion is manifested by the relativistic contraction of lengths in the direction of motion and the absence of that change perpendicular to the direction of motion, that is, by inhomogeneity. It is similar in the *gravitational* field. Unlike satellites, which fall freely in their own weightless state during inertial motion, the gas in the stationary room is drawn by gravity.

It is dense lower because of its weight, which is a relative decrease in entropy, from the satellite point of view. The satellite would spontaneously enter a state of greater entropy if it would see relative entropy in higher gravity higher (an objection to a theory that would consider relative entropy greater) and would simply abandon its inertial motion and fall into a stronger field.

We see the consistency of classical laws of physics with the (new) theory of information everywhere. The scattering rather than the merging of photons and electrons in Compton collisions is classically predicted by choosing the coordinates where the resulting electron is stationary. This process, viewed backwards in time, would give off the emission of photons (energy) from the still electrons which then leave with higher energy (for kinetic) which is impossible according to the law of energy conservation. Analogous to information theory, resting electrons do not emit information because of the principle of minimalism. On another

[20] Arthur Compton (1892-1962), American physicist.

occasion, we will see that this situation clarifies some other differences, notably complex and elementary systems, also from the point of view of information.

Compton scattering is consistent with Heisenberg's *uncertainty relations* (1927), and those with information of perception. Specifically, by shooting the still electrons with stiff rays of light (shorter wavelengths), we determine their position more precisely, but the momentum scattering is spreader, as well as vice versa. As we strive for greater determination of electron momentum, we observe them with less precise photons positions (longer wavelength). The smallest products of these uncertainties, momentum and positions (as well as energy and time), are order of magnitude of the Planck's constants and represent the smallest physical actions.

If the electron momentum is a "restriction" and the position is "capability", Heisenberg's product of uncertainties is the sum of "freedom" in the total of "information of perception". She is the backbone of the (new) theory of physical information, and here I just mention it.

Archimedes' "ability" was contrasted with the "weight" of the problem of testing the gold crown when he found that a body submerged in a liquid was easier by as much as the weight of the displaced fluid. Archimedes checked the intact crown of King *Hiero II* of Syracuse's that it was made of pure gold and that the jeweler did not deceive the king using silver, which was cheaper.

The product of two variables is again a component of "liberty" as in the case of the uncertainty relations, but defiantly formed into larger overall "information of perception" than that in the case of the principle of least effect. So we say that Archimedes has a higher "vitality" than the particles of physics themselves.

It would be a wonder if future scientists do not notice such a mix of their profession with methods of theory of information and do not intend to use it. In the end, like geometry and algebra, correct theories agree not only with their own parts but also with each other.

`http://izvor.ba/`
August 16, 2019.

1.7 Feynman diagram

The famous *Feynman*[21] on whose textbooks of quantum mechanics were raised generations of top physicists is the author of the diagrams named after him.

He was so proud of his simple and absurdly effective sketches of quantum world interactions that he put them ahead of all his other accomplishments. That Nobel Prize winner in physics was drawing them on a van to go fishing, on walls, on T-shirts, admiring their incomprehensible accuracy.

We reduce the interactions of quantum mechanics to the elementary particles that are divided into *bosons* and *fermions*. The former include carriers of forces (gravitational, electromagnetic, weak and strong), and the latter are particles that are affected by these forces.

Spin of the first is integer, the spin of the second is always half. Two or more bosons may be in the same quantum state, and this is not possible with fermions. This restriction, *Pauli's exclusion principle*, makes the behavior of the two types of particles significantly different. In the Feynman diagrams, all fermions are represented by the full line, the Higgs

[21] Richard Feynman (1918-1988), American theoretical physicist.

boson dashed, the photons, the W and Z bosons extended, the gluons usually a wavy line with feedback curves.

Electrons (negatively charged fermions) create an electric field around them, constantly emitting *virtual photons*. When a virtual photon from one electron struck into another electron it becomes real and due to the *law of conservation* the energy and momentum are transmitted. Like free boats on still water that will drift away when we throw sandbags from one to the other, electrons repel.

Thus we describe what we see in the corresponding Feynman drawing. Quantum mechanics, like classical, but in contrast to statistical, in its equations allows *inversion of time* and the interpretation of the electric attraction of electrons and *positron* (anti-electron) in the opposite direction of their times with the corresponding Feynman's thumbnail.

In the nucleus of the atoms are electrically positive *protons*, which are also attracted by negative electrons and repelled by positive positrons, so these sketches are successfully transmitted further to such cases as well as to all other known interactions, with the interpretations that gets weirder. With that comes the question of fitting information theory into these notions.

When the second electron receives the virtual photon from the first one, it receives information. The space toward the first electron becomes more informative and it spontaneously "runs away". Nature does not like information even though all of it is created. Because of this principle of information minimalism, the second electron repels from the first, but it also emits a field of virtual photons for similar repulsion.

It does not communicate everything with everyone, but here it is possible to transfer spin $+1$ photons from the first spin electron $+^1/_2$ to the spin electron $-^1/_2$, when the first one stays with spin $-^1/_2$ and the other becomes spin $+^1/_2$, in addition to some combinations that are not possible.

When a particle travels empty space, it "talks" to *vacuum*. It is an opportunity to release the excess information created by the accumulation of its past, but also a necessity, again because of the principle of information. Thus, the photon fills in and empties information, in addition to perhaps some other ways of "rippling the void", making those surpluses and deficits interesting (visible) to different participants.

Where an electron sees a surplus of information, the positron sees a deficit and vice versa, so after interacting with the same photons, the electrons are rejected by the electrons and attracted to the positrons. This is in addition to explaining the Feynman diagrams.

A simple calculation[22] shows that it is not possible to simply repel electrons by bouncing photons between them like a ping-pong ball. It will disagree with the *Coulomb*[23] force (the law of decreasing electric repulsion with a square of distance). A correct calculus will give the spherical propagation of a virtual photon, sphere by sphere from the first electron, whereby the interaction of these spheres with the second electron is a random event (not a necessity) of the chances of a decreasing square radius. These spheres are surfaces and the information is *two-dimensional* – I will try to explain it on one occasion[24].

Like concentric circles of waves of ever-smaller ripples as they expand across the surface of the water, the *amplitudes* of virtual spheres will decrease with the area of the sphere while their wavelengths remain unchanged. The decrease in amplitude indicates a decrease in the probability of interaction, and a constant of wavelengths about the unchanged position

[22] see [4], figure 1.14 and text

[23] Charles-Augustin de Coulomb (1736-1806), French military engineer and physicist.

[24] see later in this book

uncertainty and the potentially transmitted momentum.

After interacting with another electron, when the virtual photon moves from a spherical state to a particle and becomes real, all the surroundings of the first electron become unsaturated. This lack of information is filled from the first electron, or, for example, encourages the transition from the fictitious to the real state and the interaction of the virtual spheres of the second electron with the first, accelerating the opposite direction of the same interaction. This synchronization of the transfer of energy and momentum between electrons takes us one step closer to a "phantom" phenomenon, as the Einstein called it, who discovered it in 1935.

The *entanglement* of quantum interactions of dependent random events could also occur here. If we add it to the explanation of the Feynman diagrams, the first electron does not react to the presence of the second until the second photon of the first catches up with the second and is realized, and then, only then, a mutual interaction occurs. This would be an *retroactive* exchange of energies, momentum, and spin of two electrons, an alternative to the probabilistic explanation of the compensation by alternating exchanges of virtual photons.

Due to the great importance of this phantom quantum "coupling" (entanglement) for the theory of information, their connection to information physics, unknown jet, I may say something about it in another time.

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1.8 Schrödinger's cat

Schrödinger's cat is one of the most famous legends of the modern quantum world, stories that can always something be added to. It is conceived by *Schrödinger*[25] for the discussions at *Copenhagen* to emphasize the difference between the phenomena of micro and macro physics.

The cat is in a box, it is said, in which a random event can kill it, and then we open the box and find out if the cat is alive. The paradox arises if we consider that in the previous state the cat was both alive and dead or neither alive nor dead, and by knowledge she becomes only one of the two.

First of all, it is about bringing the laws of small size physics to our size. Like when an ant, which can lift a load 50 times heavier than itself, we imagine a big one like an elephant and then fantasize about an elephant carrying more than 20 tons with ease. This is not possible because the mass grows with the cube of body height, the area with the square, and little of it is linear with length. It is in our nature to admire the colors carelessly, but in the nature of the little one, even a collision with the smallest piece of light would be fatal to us. Shrinking your body changes your physical characteristics.

When, after Plank, Einstein, and a few other geniuses, in 1924, *Louis de Broglie*[26] emerged among quantum mechanics theorists with the idea that all matter has wave properties (interference, diffraction), dominant in the micro-world, Schrödinger first penciled in and defined the mathematics of such a hypothesis.

Soon, in 1926, he coined his famous wave partial differential equation and defined de Broglie's fantastic world. Surprisingly, the solutions to his equation began to be confirmed

[25] Erwin Schrödinger (1887-1961), Austrian physicist.
[26] Louis de Broglie (1892-1987), French physicist.

experimentally, one by one without exception. Schrödinger, with his discovery of quantum mechanics and an army of experimental physicists in the laboratories by which they had until then wandered through the dark, provided an astonishingly powerful theoretical tool.

He was aware that his theory was ahead of practice, that concrete examinations would only confirm him, and he was in a hurry to go as far as possible in the new field. In many of the scientific papers he published at that time, he successfully interpreted the derivations of his equations, but perhaps he struggled most difficult with the paradoxical *superposition*, then a new phenomenon and an indomitable solution. It is the most common place where the interpretation of quantum mechanics is accelerated and needs clarification.

Quantum mechanics is so mathematical that it is confusingly accurate and precise in experimental predictions, and certainly because of the same, it is also non-contradictory. It is a representation of the first differential calculus (Schrödinger), then the matrix (Heisenberg) and unitary spaces (Hilbert). She is all about analysis, algebra, probability theory, always with the same concrete endings, and that are exactly the same. It's normal for mathematicians and we won't talk about it.

What is seemingly strange is the theorem of algebra that any polyvalent logic (true, perhaps, false) can be reduced to divalent (true, false), and what now with the irreducible state of superposition? Just seemingly, like six states of pre-roll dice that after throwing always, but always collapse (quantum mechanics term) into just one of them. In addition, the moment of collapse is "elusive", so this also bothered Schrödinger.

The Copenhagen interpretation, formulated by Bohr and Heisenberg in 1927, is today the standard interpretation of quantum mechanics. It dismisses a question like "where a particle was before measuring its position" as meaningless. The act of measuring the superposition of multiple possibilities is realized in one outcome, that is, information emerges from uncertainty – I would add.

The uncertainty is reduced exactly by how much information is generated, and the process is violent, triggered by measurement, interaction with measuring devices. These are IT interpretations. First comes from the law of conserving the information, which is why uncertainty is the type of information, and secondly from the principle of minimalism of nature around changing the state of information.

All matter is made up of data only, and we measure their quantity with such information for which is conservation law valid, hence quantum validation. Namely, unlike infinity whose proper subset can be a quantity equal to the whole set, every property of information, due to the law of conservation, is finally divisible.

Furthermore, each finite set has finally many combinations, and similar causes lead to similar immediate consequences (not necessarily to further), so the information phenomena of physics are always periodic, but vice versa, each periodicity of occurrence has something additional to its information. Matter consists only of information and here is an explanation of the Schrödinger wave equation phenomenon.

Finally, information is the equivalent of an action, at least the quantum of action (the product of the uncertainty of energy and time) and therefore – if a cat is of great mass, it is of very high energy, so the transformation of that energy is a very short-lived phenomenon.

She has nothing to remember when we see her in the box, because her process of collapsing from a superposition lasted a very short and there is no means in measuring something so short-lived, wither to "experiencing." Only a very light "cat" would have a longer uncertainty of the past, so that we could speak of the phantom action of the present onto the past, that is, the *retroactive* action mentioned in the previous story.

Let us return to the classical interpretations of quantum mechanics. Copenhagen's is

bemoaned that "the Moon will not disappear if we do not look at it!" (Einstein's argument), and then there are questions of the objectivity of a multitude of uncertainties (superposition) before "looking" into appearance. That is why *Everett*[27] in 1957 formulated the *multi-world interpretation* of quantum mechanics. According to him, both living and dead cats have become real, but in separate realities, there is no effective communication or interaction between them. In the informatics, both of these interpretations are now two parts of the same theory.

There are different hypotheses, and new ones are constantly emerging. The "informatics interpretation" mentioned here is one of the candidates.

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1.9 Twin paradox

The God hypothesis in science is not needed. Unless we say, "God is the truth." After which atheist mathematicians would be in trouble. They might say: God like to hide, but he does not like to be hidden. This is supported by the principled minimalism of information and in dual the objectivity of at least some uncertainty. Also, it is in the difficulties and paradoxes of science, and on the other hand, in our successes in it.

Instead of the usual critique of the *relativity* based on its paradoxes, we do the opposite here. We will note that it has been experimentally confirmed up to almost a tenth decimal of a pound, a meter and a second. We will admit that the theory of relativity after quantum mechanics is some of the most proven that physics has today, and that in this sense it is the top science in general, and then look at what its paradoxes further tell us. And among the first is the famous *twin paradox*.

Einstein's Special Theory of Relativity was published in 1905. It is postulated that all motions are relative and that the same laws of physics apply to each of the inertial systems observers. Another postulate states that the speed of light in a vacuum is constant, independent of the speed of the source. The consequences are universal proper time (resting observer) and slow time flow in the relative system (motion observer).

Now imagine two twin brothers – one who stays in the first system (on earth) and the other who staggers equally straight and, after some traveling, equally returns. When the two brothers are together again, the paradox says that whatever one of the brothers is still, the former is older than the other and the latter is older than the former. But when they find themselves in the same place again, it will be seen that both claims are not possible!

Einstein himself once gave a solution to this situation, saying that the passenger had to rotate, slow down and accelerates to back, disrupting inertial movement. Because one of the brethren was leaving a single movement there is no equality of their conditions and no contradiction mentioned. We will not doubt this explanation, but will use it even further.

As the second system moves away from the first, relative time flows more slowly, lagging behind in the past compared to the present. After slowing down the speed, to turn and rewind, again because of the slower flow of time, the second system is then constantly in the future of the first. The difference between the two presents is diminishing until the alignment of the origin and the meeting of the brothers when that difference disappears. Part of the traveler's own time the relative observer from the ground does not see. The total

[27] Hugh Everett III (1930-1982), American physicist.

elapsed proper time has been longer than that of the relative for the part which the relative observer does not see, which in relation to him, we now say, has gone to *parallel reality*.

However, the slower relative flow of time produces *virtual* energies (vacuums) into real ones that are not visible to the proper observer. As time goes on, the product of energy and time uncertainty grows, some virtual particles become real, with them virtual actions as well as virtual information. The relative observer now perceives just as much new virtual information as the part of the proper has been hidden in "parallel reality".

The theory of relativity, Heisenberg's relations of uncertainty and the law of conservation information are all in sync. But this interpretation is just beginning here.

The disappearance of part of one's own information in "parallel reality" with respect to various relative observers can take place in three *dimensions*. Specifically, in each individual Minkowski space-time plane, otherwise Einstein's geometric basis, the relative time axis tilts in the direction of motion. Since there are three dimensions of space, there are also three dimensions of motion, and tilting of time axes is also possible in three dimensions, so there are as many times dimensions as there are spatial.

This multidimensional nature of time, which is otherwise characteristic of (my) information theory[28], is a novelty in physics. Note that different dimensions of time are not considered even in *string theory*. It is still a hypothetical branch of physics that unites known forces and treats space on ten dimensions, but with a constant one and the same axis of the time.

In principle, *hiding information*, now revealed in "parallel realities", is also seen in the more frequent realization of more likely events (less informative), in easier coding than in decoding, in spreading lies faster than truths through social media and networks. On the other hand, the existence of "parallel realities" follows the existence of objective *coincidences*, and from this the theory of information, so that these three (hypo) theses are also in agreement.

In the end, this is just another of many reaffirmations that "the truth likes to hide, but does not like to be hidden".

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September 6, 2019.

1.10 Bernoulli's attraction

Any three correct theories are as non-contradictory with their parts as they are with each other. We know how to transfer geometry theorems to algebra and probability or vice versa, but it is instructive to discover this on *fluid* (liquid or gas) dynamics, relativistic motion, and the principle of information. A few years ago, one such discussion was imposed on me by a colleague, suspecting that the branches of the natural sciences can be easily connected as we do in mathematics.

The main part of that discussion is in my then-published book, Space Time [8], and here I will try to convey some interesting details and perhaps at least some new untold discovery. The idea of the connection that came to me came from a little-known *train paradox*, from somewhere as a purported proof of falsity of theory of relativity. Of course, we will not pay attention to the fun "wise deductions" (to me by an unknown author), but we will use this interesting initial construction to look a little further into the theory of relativity.

[28] because of the postulated objectivity of options

The story goes like this: two steadfast equal-speed trains pass by. The space between them is so open that the air can flow freely. We know that flowing fluid sucks the surrounding substance. This is also said in the equation of 1738 *Bernoulli*[29]: the attraction of fluid grows with the square of its velocity, besides some other irrelevant magnitudes here.

This means that the observer from the coupe of the first train should notice the movement of his air towards the second one, and contradict the observer from the second train to claim symmetry — to observe the opposite movement, the air coming out of his wagon. Consistently further, for the outside observer standing on the embankment next to the trains, two air movements, from wagon to wagon, are canceled.

Bernoulli's equation is indisputable. It is easily proved by the flow of fluid through a tube of different profiles (and therefore the speed) at which the openings with pressure gauges are laterally mounted. We also see its application in the aviation industry in the construction of *airplane wings* with a longer upper profile that airflows faster than lower ones, which makes the *thrust* of an airplane upward with the square of the airplane speed. The thrust also depends on the wing surface, aerodynamics, air density, but we can now ignore such parameters.

The special theory of relativity is also indisputable. It is derived from only two well-checked principles, relativity of motion and constant speed of light, and with its consequences we improve the operation of various technical devices. For example, GPS (general positioning system) starts from satellites that calculate the contraction of lengths and time dilation first because of inertial motion in orbit and then because of the gravitational field. This effect is exactly proportional to the increase in the kinetic energy of the body in motion, which at lower speeds (relative to the speed of light) receives the long-known value of the product of half the mass and the square of the speed of the body. Surprisingly, we add here, the same proportionality holds for the Bernoulli equation!

The wagon is relatively shortened only in the direction of movement, and its volume decreases exactly as many times as Bernoulli's attraction increases. Therefore, these two effects are canceled out because the increase in compressed air pressure outward is in equilibrium with Bernoulli pulling inwards. In other words, we have just found another derivation of the famous Bernoulli equation, now using a special theory of relativity.

The next question is: where is the principle of information? See first my previous post about the twin paradox. In a relative (moving) system, time flows more slowly than in one's proper (own, immobile), so virtual particles (energies, actions and information) become real. They void the exact amount needed to cover the loss of information due to the departure part of the moving system into *parallel reality*. We can say that the slowing of time pushes a part of the system into parallel reality (inaccessible to the relative observer), which due to the law of conservation draws *virtual information* into the real (inaccessible to its proper observer).

Theoretically, if layer by layer we were to have faster movements of concentric spheres as we approached their common center, the effect of the central gravitational field would appear, at least with respect to time deceleration. Bernoulli's equation states that then there would be *suction* of the substance towards the origin of the spheres. So slowing down time is the equivalent of the attracting, and since all matter is made up of information, what we have here is "sucking in information."

The same phenomenon becomes more familiar if we look at things in reverse, on a flat surface that rotates about one point. Beyond the axis of rotation, the tangential velocity

[29] Daniel Bernoulli (1700-1782), Swiss mathematician and physicist.

(perpendicular to the radius) increases, so the pressure of the substance outwards increases, now we can say the Bernoulli pressure. However, these 'suction' forces are already known to us as centrifugal, and we know that they exist even when there is no (visible) substance. *Centrifugal force* is the force of space and time itself! It has an equal effect on space time and matter, forms of information, which is why it is actually a phenomenon of information itself.

If the vacuum did not have virtual effects that could translate into a real by time slowdown, then the law of conservation (the amount of) information would be violated, or *gravitational force* would have to be much stronger. In principle, the lack of information is as attractive as the excess is repulsive.

In this way, fluid dynamics, relativity theory, and the principle of information seem to be parts of one and the same, for now say unknown, larger theories.

`http://izvor.ba/`
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1.11 Mechanism

The German astronomer and mathematician *Kepler*[30] was among the first to deal more accurately with celestial bodies.

From 1609 to 1619 he discovered the laws that bear his name today: first, that the planets move in ellipses in which one of the (two) foci is the Sun; second, that the Sun-planet (radius vector) strokes equal surfaces at equal intervals; third, the squares of planetary patrols are proportional to the cubes of their mean distances from the Sun. Thus began the *mechanism* era.

With them, the Italian mathematician *Galilei*[31], who, when throwing objects from the Pisa tower in 1654, discovered that all bodies would fall with the same acceleration if there was no air resistance. While others argued that movement required constant body pushing, Galileo also understood the law of inertia.

All such discoveries of classical mechanics as the icing on the cake were given by the English mathematician *Newton*[32] in the book "Philosophiæ Naturalis Principia Mathematica", printed on July 5, 1687. Displacing away from the plague epidemic of 1665, in Woolsthorpe Manor, his birthplace north of London and Cambridge where he performed most of his experiments (working on optics), he also reportedly watched an apple fall from a tree and got the idea of the force of gravity.

As a good mathematician, Newton deduced from Kepler laws that gravitational pull decreases with a square of distance. He was a mystic by nature and saw no problem in spreading force through solid bodies, or in its immediate action at a distance. Watching the water in the washbowl spill as the basin spins, he decides on the absolute space. Just doing such an experiment points to the difficulties Newton observed in Galileo's inertia, with systems in uniform rectilinear motion (inertial) in which all laws of physics should be invariant (corresponding).

The principles of Newton made a deep impression on all later explorers of nature, and their extreme ability to predict derived from formulas, precision, and in general determinism,

[30] Johannes Kepler, 1572-1630
[31] Galileo Galilei, 1564-1642
[32] Isaac Newton, 1642-1727

seemed to enchant them. Philosophical Essays on Probability by *Laplace*[33], the last of the leading 18th-century mathematicians may confirm this. They start from the realization that we need probability only because we are not well informed, and his famous sentence is a summary of mechanical materialism of the time:

– A mind that would know all at a given moment the active force of nature, as well as the relative position of all the particles that make up it, and yet be large enough to subject it to mathematical analysis, could include in one formula the movement of the largest bodies in the universe and the smallest atoms in it; there would be nothing vague for him, and he would see clearly both the future and the past. That perfection, which human reason has been able to give to astronomy, is still a poor idea of such a mind.

A deeper consideration of probability consistency would reveal the first inconsistency of Laplace's mechanical concept with his the most famous treatise (Analytical Probability Theory). Laplace discussed in detail hazard games, geometric probabilities, Bernoulli's theorem and its relation to the integral of normal distribution, as well as Legendre's least-squares theory. The striking consistency of such considerations was crowned in 1933 by *Kolmogorov*[34] discovering axioms of probability and basing it as a branch of mathematics.

There was something more in this "lack of information" that we could rely on for *probability theory* and the development of statistical physics in the 19th century, but the real shock came with the discovery of the deterministic *theory of chaos*, in transition from the 19th to the 20th century. It was discovered and based on systems whose slight initial variations gradually evolve into very different states such as metrological phenomena. Literally, the smallest grain of dust could spoil and stop the perfection of *classical mechanics*. On the other hand, it was known that the greatest stability of the system's operation could be derived from the laws of large numbers of probability theory.

It happened that none of the great researchers tried to exploit that convenience of probability that it was a branch of mathematics. As early as the 18th century, the problem of *Buffon's*[35] needle, the formula for the probability of accidentally dropping a needle to the floor with drawn strips, and the way (law of large numbers) how with the number of needle throws the result of the experiment approaches the exact value of the formula. Because the formula contains perhaps the most famous irrational number – pi (3.14159...) by increasing the number of throws, we get that number with increasing accuracy, and in similar way much more. Such harmony with mathematics has no say statistics. Conversely, the mismatch of probability and mathematics would indicate the non-coincidence of the phenomena.

So, one might devise: well, because of not being known of everything that happens with gravity, I will rely on probability theory, for it is so consistent that it will surely be in line with future findings that will be based on better knowledge and determinism. Failing that good again, I proved the determinism of Newtonian mechanics. No one came to mind it because the age of mechanics was at its peak.

It is similar to other physical phenomena of which we have no absolute knowledge. For the glass in front of us we would say: it is there on the table, because in the given circumstances it is its most probable condition and is the result of a large multitude. The most likely outcomes are, in principle, the most common occurrences, so the cup is still in the same place in the next moments until some force (hand) appears and moves it. Therefore, force changes probabilities. The diversion of satellites (bodies in freefall) from their own

[33]Pierre-Simon Laplace (1749-1827), French mathematician.
[34]Andrey Kolmogorov (1903-1987), Russian mathematician.
[35]Georges-Louis Leclerc, Comte de Buffon (1707-1788), French mathematician.

trajectory from the point of view of the satellite is less likely. This would follow from the mere agreement of probability with mathematics!

At the time of *Hartley's* discovery of information in 1928, the said principle of probability would become the principle of information: less informative is more common. We could then say that bodies fall freely, avoiding communication. But it had to wait for a century.

`http://izvor.ba/`
September 20, 2019.

1.12 Materialism

At the time of my first recent writings on gravity, I was conducting correspondence with colleagues from various professions. Those about *materialism* could then be more interesting than the main topic and, on the advice of one of them; here are the more interesting parts in the foreground. Unfortunately, I have only preserved some of these discussions, and most of them have to be reconstructed by memory.

Discussions were held around three issues. Is the *probability* a branch of mathematics and statistics are not, can relativistic gravity equations be derived from probability theory and why did Einstein not do it? Suppose the answer to the second question is positive (I has been able to provide the proof), but I may talk about it later.

Unlike various nebulosus, including natural sciences and statistics, probability is a branch of mathematics, since its proofs are transferable to mathematical analysis, algebra, and geometry. I'll explain that with an example.

We toss a *fair-coin* until the tail falls. For a tail to occur in the first throw, the probability is $^1/_2$. If it happens in the second throw, then the head is dropped first, then the letter, so the probability is $^1/_4$. If the event occurs in the third throw, a series of HHT probabilities of $^1/_8$ occurred. That tail in the second cast is not the same as the tail in the third and in general the events in this series are all independent. Their probabilities add up. Sooner or later the tail will fall, so in an infinite sum of the fractions we have all possible outcomes, which means that their sum into one has the probability of a certain event.

That the sum of an infinite series of such fractions, that the sum of half of half's is summed in one, can be proved algebraically without mentioning probability. It is such ability to transmit the proof that no statistics have, which is why we say statistics are not, and probability is a branch of mathematics. Pythagoras' theorem is not proved by experiments, and even experimental sciences are not branches of mathematics, though experiment is also a proof by contradiction, the basic tool of mathematics.

So it makes sense to declare yourself uninformed in mechanics issues and look for probabilistic settings to derive equally accurate (with classical) equations of motion. Especially, it makes sense in the everyday macro-world where this is guaranteed by the law of large numbers (again) of probability theory.

Einstein[36] would not be happy to work on shaky foundations because he was a great researcher, and they are rarely happy with half solutions. To give anything was usually not a style of such, so he sought the maximum in deterministic geometries and tensor calculus. He noticed that the satellite in the gravitational field was actually in free fall and in the local weightless state, and concluded that the space-time geometry defines gravity and that mass

[36] Albert Einstein (1879-1955), German-born theoretical physicist.

and energy define the geometry. This is the essence of Einstein's derivation of the general field equations named after him.

Translated to the *tensor* metric, satellites fall moving along *geodesic lines* that represent the shortest paths between given points, also the least possible energy exchange, and the least amount of time spent. It has recently been proven that these geodesic follow the principle of least action, and now I would add the principle of (least) information.

The search for a fundamental solution in determinism and geometry only partly explains Einstein's "onslaught" to field equations published in 1915, especially since, ten years earlier, along with a special theory of relativity, he also published an analysis of *Brownian motion*, the random behavior of particles in solution, and then much of it in quantum mechanics rather non-causal. Other reasons lie in the philosophical atmosphere of the time.

At the turn of the 20th century, mechanicism prevailed in the natural sciences. The belief that *physics* should become independent, that matter is one and abstract ideas are something else, that the natural sciences should stick to the sensory things, was breaking through. This was favored by the fact that mathematics grew into an increasingly incomprehensible "story" for many. The social sciences imposed *Marxism*, dialectical materialism, and Plato's former world of ideas was losing the race.

One of the leaders of such philosophy was *Mach*[37], one of the leading physicists of the time. It is said that on one occasion he came to Boltzmann's lecture on the propagation of heat by vibrating molecules, stood up, turned to the audience and exclaimed, "People, don't listen to this man, this man is a fool, atoms don't exist!". Boltzmann committed suicide soon (he was prone to depression) but his theory prevailed.

Stricter to look at and geometry would be problematic in materialism. The substance world nowhere builds such straight lines, much less so thin that they could be justified in construction, and then we would have to renounce his human creations and stick only to the primal, not to say instinctive or animalistic. The philosophy of mechanistic materialism came to an end already in Einstein's general theory of relativity.

There are other times today. I advocate a philosophy of nature according to which information is everywhere. They come from "local" unpredictability. That's why we communicate, because we don't have everything we need. That's why I'm talking about "perception of information", because every local world (particle) is a world in itself; it's legal in that theory. It is irrelevant whether it is leaning on one broader uncertainty, on the general unpredictability generated by endless abstractions such as mathematical ones, because that "world of ideas", that is, the world of truth, is always greater than imagined.

We can say the latter freely because there is no set of all sets (Russell's paradox), no theory of all theories (Gödel's incompleteness theorems), and no ideal criterion (Arrow's impossibility theorem). In this context, Einstein's general theory of relativity has met expectations, but is also ripe for new content.

`http://izvor.ba/`
October 4, 2019.

1.13 Space and time

Space, time and matter are types of information. They exist and are objects of communication. It is in the spirit of nature not to communicate everything with everyone, so

[37] Ernst Mach (1838-1916), Austrian physicist.

the logical question is whether there is "something" that our space-time does not directly interact with, and that such something also "exists".

To the proponents of mechanistic materialism, such a question would be pointless, because what we cannot touch, smell, see, and feel at all, which cannot directly affect us, we would say, does not exist. But they would not be right.

The great German mathematician *Gauss* (Carl Friedrich Gauss, 1777-1855) once dealt with a similar problem whose solution he published in 1827 under the name "exceptional theorem" (Lat. Theorema egregium). He asked himself if our everyday space was flat (Euclidean) or maybe it was curved (non-Euclidean) and whether and how we could find it out. Following in the wake of Gauss's solution, his student *Riemann* found mathematical forms of non-Euclidean geometries, and using them *Einstein* came to his general field equations.

We will not delve deeper into the entanglements of non-Euclidean geometries, as Euclidean geometries might be too difficult for us, but we can still understand some of it. An ant walking on a sphere might "notice" the finality of its surface as opposed to the ordinary plane. It would continue along the shortest paths between the given places, but these paths on the sphere would be the largest circles, the geodesics of the ball surface.

The curvature of the surface can be considered as follows. Put a vector (oriented along the north) on the north pole of the Earth, which then moves (translate) in parallel with the zero meridian, past London, to the equator, an imaginary circle that orbits the Earth at an equal distance from the poles. We drag the vector further in parallel with the equator to the meridian of Belgrade, and then lift it in parallel with that meridian all the way to the North Pole. The starting and ending directions of that vector in the north are not equal. A direction defect is a change in information (uncertainty).

The disturbance of the direction is also equivalent to the change of the momentum vector. The change of impulses is caused by the consumption of energy (work), and it is caused by some force. However, representations of the vector change could not occur in a plane inertial space, so the aforementioned translations prove in sequence: that the space is curved, that it is a field of some force, that it has information.

Also, the method reveals additional *dimensions of space* in which the given Riemannian space is curved. As curvature means the presence of gravity (theory of relativity), and then how the direction of gravity with the time axis (time of travel the speed of light) defines a plane, at different points of the field different, whereby the time axis tends to be more inclined towards the spatial where the gravity is stronger, we find that there are three *time axes* due to three spatial ones.

It is, in fact, the (unknown) direct consequence of Einstein's theory of gravity, which tells us about six dimensions of space-time. Additional spatial (not temporal) dimensions are also addressed by contemporary physicists within string theory, but this is not the case here. Their theory is basically deterministic, and here we are pointing out information content.

I'll reinforce that story with *topology*. Topology is a part of geometry stripped of the definition of length. It may seem that it is even more deterministic and further from the concept of uncertainty, but it turns out that it is not. The inductive topological definition of dimensions is particularly convenient for this.

A point is of dimension zero and any finite and discrete set of points is of dimension zero. If a figure (set of points) has a given dimension, then a discrete set of the same figures has the same dimension. If the "border" (set of points) is of a given dimension and it separates the "interior" from the "exterior" of some "figure", if it completely separates the two areas, then that "figure" has a dimension one larger than the "border".

For example, the boundary of leg (length) is two points (dimensions zero), and the leg is a part of a straight line, so the straight line dimension is one. It is similar to a circle or some other curved line. The boundary of an interior of a circle is a circle line of dimension one, so a plane has dimension two. The same applies to a sphere or other curved surface with a closed (curved) line. A sphere (dimensions two) isolates the interior from the outside of space, so the space is dimension three. If the present, with all its 3-D space at a given moment, separates the past from the future, then the space-time dimension is four. Such is the nature of absolute, classical space-time.

In the special theory of relativity, inertial systems of straight linear motion, the concept of simultaneity is relative, but it can always define the "present" which completely separates the "past" from the "future", and so the individual has dimension of the four. This means to us that inertial systems are minimal communications, but more of them are no longer such. Different directions of inertial motions tilt the time axes differently and with three spatial dimensions build three temporal ones, the same as in the mentioned gravitational field. Let's look at this result once again in a way that confirms the connection of additional dimensions with unpredictability or information.

Imagine a walled prison cell and its duration. Without uncertainty and erosion, it would be a 4-D building that could insulate the interior from outer space-time indefinitely. In the case of erosion, the isolation of a part of space-time (figure of 4-D) is not infinite, so points (events) from inside the cell would sometimes be outside and, according to the topological definition of a dimension, would form a space-time of dimension larger than five.

I sketched some of the ways of discovering reality within 6-D space-time, which at a given moment of communication is always within only four dimensions. That is by the very methods of mathematics that would be criticized and successfully challenged by supporters of mechanistic materialism. This is, among other things, the greatness of the genius Gauss and his student Riemann, who lived in a time of growing influence of such a philosophy and did not quite succumb to it.

`http://izvor.ba/`
October 11, 2019.

1.14 Principle of information

There are coincidences, but nature does not like them. It is the *principle of information* in the narrow sense: information is inevitable and shy.

That's why we work so hard around various predictions and probably make mistakes, because we work with objective unpredictability that interferes with every business. It is a matter of principle to skimp with information around us with its gentle but persistent tendency to hide knowledge and truth wherever it may be hidden. We rarely notice it.

For example, *journalists* rightly say that the news "a man has bitten a dog" is greater than the news "a dog has bitten a man" because it is less likely. For the same reason, the "tails dropped" information when throwing a fair coin is greater than the outcome information of the "tails" of non-fair coin if it is expected. There are principle shifts from this "wasteful" situation also in other ways.

We consider fairness of the initial conditions of participants in the *sports* competition, and they maximize their chances. Increasing the number and success of individuals' bidding options release the effort to raise the quality and fierceness of the fight, making the event richer. The principle of information is to suppress these shifts. This is because information

is a physical action, equality increases the chances of conflict, and conflicts are unfavorable, they are arduous.

It turns out that equality is a fair, useful and unpleasant condition. Equals have more chances for advancement, but also indivisible goals and reasons for conflicts, so we have come to terms with regulating society through equality-based legal systems.

Creating equality creates prerequisites for new *conflicts*, which makes the system self-validating and growing. *Legal* regulation spontaneously becomes more complex, denser and more expensive. It gradually diminishes the freedoms of individuals by stealing them for themselves, and then suppresses fair conditions. Excessive organizing the society for freedom and equality leads to non-freedom and inequality.

Social phenomena are not exempt from the law of information and then from *duality*, its important traits. Uncertainty is the essence of information, the source of observations from impossible to certain events (right ahead to theorems), because certainties are knowledge too. Nature runs away from uncertainty to certainty wherever it can, but the latter are so little informative that we do not perceive them directly but understand them abstractly. Everything we can experience, and then space, time and matter, but also the sure things of the species are information.

Examples of minimalism include *feminization*, that is, spontaneous growth of *entropy*. The gas molecules in the room are distributed uniformly because this arrangement is more likely than crowding. We call the inside order a mess because we look at it from the outside. The things that are arranged are less outwardly informative, like uniformed and lined up soldiers at the parade, they are impersonal and amorphous. The aggregate information of the room and its large enough environment tends to remain constant.

That the order emits less information seems controversial (to my colleagues), so I give another example. Text made up of random words in a dictionary is less a "message" than a real messaging. In the contained text, some words are statistically significantly more frequent than others; their frequencies of occurrence in the text of the message differ. I appreciate that zero order, that is, absolute *clutter* is impossible (Ramsey's theorem: enough random words will contain every pre-given sentence), moreover, I will note that this is also in accordance with the principle of information.

The spontaneous growth of entropy is recognized in the second law of *thermodynamics* (the transition of heat from a warmer body to a cooler one), but it is also evident in the consumption of each individual *society*. Successful civilizations are filling their deficits of comfort and safety by running, developing, say civilizing, with more and more internal order and less external aggression.

It's similar to the forms of *life* in general. They arise by complicating and accumulating information with its principled thrift, and then because of the same, the body sees them as surplus that a living individual resolves by interacting with the environment or transferring his or her own action (information, freedom) to a higher form in the organization.

Together with matter, space retrieves and accumulates some of the information of the present. The increasing entropy of the substance of the universe reduces the emission of information exactly to the amount of accumulation in space. *Vacuum* is a warehouse of the past that is constantly growing.

The obscure action of diminishing information is also evident in the *law of large numbers* probability theory. As we move from the micro-world to the macro-world the complexity grows, the uncertainties become certain, so in the "world of the great", instead of having more information we recognize it less and less. We also discover these kinds of hidden accumulated information through "restless" forms of *distributions* (bell-shaped, exponential,

degrees), always because of the inevitable coincidences and the tendency of nature to negate them.

Because more informative events are less likely and because nature loves them less, social misinformation networks travel faster. Lying is more attractive than truth, fiction than science, decoding as an act of knowing is harder than coding.

The action is information, but the strike of a stone throw to the head does not made you more informed thanks to the principle minimalism of information. On the other hand, the development of human knowledge over the millennia tells us that the nature its truths fail to hide from us.

`http://izvor.ba/`
October 18, 2019.

1.15 Dimensions of time

The concept of adding *time dimensions* is one purely mathematical abstraction and will never have anything to do with reality – is one of usual questions to me – because there where is no information it does not exist, and there is no emissions of (physical) information between *parallel realities*.

I will note that in such questions the "mathematical abstraction" is dehumanized and the mechanistic philosophy is bribed, but let's start in order.

If the concept of *simultaneity* were universal to all physical systems of the universe (bodies in motion, gravitational fields), then we could represent all 3-D space with a single point on the time axis. If that point of "outer space" is far from the origin it would indicate an older universe, and that's it. We wouldn't know what's inside. There is no information on the layout and dynamics of matter, which is why the model *information universe* we seek should look different.

It may be a surprise, but *theory of relativity* is also[38] a good start, although it is seemingly exemplarily deterministic, and options and objective coincidences are important for the "IT universe." Due to the relativistic prohibition of synchronization of all the presents and the inability to define a unique 3-D space of the whole universe, the assumed point on the time axis grows and degenerates in its course. It expands in a 3-D coordinate system of time as each new direction of motion seeks a new slope of the time axis higher with the speed of movement.

Formal replacement of space and time variables is possible. There is an analogy of expansion and the same number of dimensions of time as space, each of the three, so that the transition of "temporal" to the known spatial-temporal model of Minkowski's theory of relativity is formally possible. We recognize heaps of light touching the cone vertices at the origin. However, the growth and expansion of the "point of space" further speaks of freedom of options, of the various possibilities of evolution of the universe in accordance with assumed objective coincidences. Here is an example, and then I keep going.

Biological species in *Darwinian evolution* also have a breadth of development. The great variety of ways of adaptation to the environment for the survival of individuals, as well as ways of extinction, indicates that there is often no best path here, besides some evolutionary convergences such as the formation of the same physiology of the eye from different starting points. There are various triggers for change.

[38] proofs of this thesis I have various

For example, the development of the power of a lion, deer antlers, colorful feathers in some birds or the intelligence of humans has also involved courtship. The sexual attraction of a trait is an option, in addition to the bare ability to survive. The emergence of intelligence in humans may have been more risky than useful, because the brain knows to spend more than to contribute, but it survived.

So we could also look at the *evolution* of the inanimate world (physics) and explain one of its problems with the spontaneous growth of entropy (the second law of thermodynamics). Paradoxically, in principle, scarcity with emissions of information tends to evolve physical states into less informative ones, so then it seems that a reverse flow of time is not possible. But this would then be inconsistent with quantum mechanics, because all the evolutions of quantum states are defined by unitary operators that are reversible and allow the opposite flow of time. An explanation of this paradox, among other things, is possible from the standpoint of *cardinality* (infinity) of set theory.

The consecution of moments in which we exist, as well as the set of (all) atoms of the universe, may have at most *countable* infinite elements. There are so many natural numbers, there are so many integers, there are just as many as all fractions, we say there are *discrete* (moderately) infinitely many. Due to the law of conservation information and the characteristics of *infinite* sets that they can be equal (in quantity) to their proper part, the information is finally divisible, so let's say discrete sets. Their union as a discrete set of discrete sets, a universe of information, is also discrete.

However, there are many more real numbers, countless infinitely many, say *continuum* many. We generate them in a series of many positions by varying the values of the positions of the members of the array, allowing two or more options for an infinite subset of those members. The catch is that the continuum is so much larger than the discrete set that the probability of choosing a fraction, *rational number*, on an arbitrary part, the interval of real numbers – would be zero. Moreover, in (countably) infinitely many attempts, the probability of choosing at least one single fraction is zero.

In other words, randomly choosing a rational number among real ones is an "almost impossible" event. Practically, in a universe driven by coincidences, the chance that same one *antiparticle* (particle with reverse time flow) from our present could have come to an instant earlier (in our past) is an impossible event. This also means that the antiparticles that we see now, and then in the next moment, are never the same.

This is why we need a continuum of 6-D space-time in a universe of information whose every property is finally divisible. That's why Minkowski's 4-D space-time is a continuum. Therefore, the assumptions of the infinitesimal calculus are valid so that the equations of motion of physics, Einstein's equations, or the Riemann geometry (due to the infinitesimal calculus) can be correct theories.

There are many similar "real" consequences of the aforementioned "mathematical abstractions", but the *reality* for unconscious beings such as grasses, ants, or some larger animals, does not make sense of what reality is. We also are still searching for these true determinants of reality, and we should not rush and say "it is not realistic" for something that, for example, we cannot eat.

`http://izvor.ba/`
October 25, 2019.

1.16 Doppler effect

Christian *Doppler* (1803-1853) was an Austrian mathematician and physicist, best known for the effect that bears his name and which represents the relative change in the wavelength and frequency of the source wave in motion relative to the static observer.

This seemingly innocuous occurrence of the thickening of the waves we approach and the dilution of those we depart from has unexpected depth and interesting implications.

The Doppler Effect or shift also occurs in the *sound* of the siren of a vehicle passing by us, which has a higher tone on arrival and lower on departure than the same sirens at rest. Wave velocity is the number of oscillations multiplied by the wavelength, so in a medium (air) of constant wave velocity (sound 346 m/s at 25° C) this means that the waves of the source approaching us are shorter and longer in departure. The younger person's aural perception frequency range is from 20 to 20,000 hertz (blinks per second).

It's similar to the *light*. As the Earth in orbiting the Sun approaches or moves away from the fixed stars, their color apparently shifts to purple or red, in the first case to wavelengths of about 300, and in the second to 700 nano meters. The frequencies of these edges decrease from 1,000 to about 430 terahertz, so that their product with a wavelength is equal to the speed of light in a vacuum, otherwise independent of the speed of the source.

The special theory of relativity confirms this effect, but also adds a transversal (lateral) decrease in frequencies in proportion to the relative slowdown of the flow of time. The arithmetic mean of the longitudinal (lengthwise) frequencies of the light source at arrival and departure is exactly equal to the transverse frequency. Due to the constant speed of light in vacuum, the corresponding decrease in frequency is an increase in wavelength. The general theory of relativity by consistently slowing down time in stronger gravity predicts the slowing of light frequencies and again an increase in their wavelengths.

According to quantum mechanics, this wave elongation could be interpreted by increasing the uncertainty of the particle-wave position, and according to the (hypothetical) theory of physical information, we see that the positions of the space we approach are more likely than those of which we are moving away. Moreover, in accordance with the treatment of the (6-D) space-time of that theory, we can say that we are going to the future because it is more likely for us.

I will repeat, the relativistic effect, that the inertial system (body) approaching us has a slow flow of time whereby its and our present will equalize at the moment of encounter, meaning that it is until that moment in our ever-closer future.

On the other hand, an inertial system that is moving away, and also has a relatively slower flow of time, is lagging further in our past. Interpretation of higher probabilities due to shorter wavelengths of the first system and inversely less probabilities of departure positions is the new in the theory.

We have the final result in both, relativity theory and quantum mechanics, and it relates to 6-D space time (I described earlier). This explanation would not be possible without the "absurd" relativistic contraction of lengths in the velocity direction (or in the direction of the field) at the same time as the increase in wavelengths.

The seemingly harmless Doppler shift has some more interesting implications. Let's see what it has to say about *entropy*. It is known from classical and relativistic mechanics that the (kinetic) energy of a body grows with speed, but Planck's quantum formula says that the energy of a photon is proportional only to its frequency. It is paradoxical that the Doppler effect predicts a decrease in this energy due to the movement of a light source that increases energy.

One possibility is therefore to redefine the terms *heat* and body temperature because of the famous Clausius fracture, the quotient of heat and temperature, which he called entropy in 1850. We declare that the movement increases the *temperature* of the body and its kinetic energy but not the heat.

We observe an increase in temperature as a Doppler shift towards red, and the resulting decrease in entropy is consistent with Boltzmann's statistical interpretation. Namely, the relativistic contraction of space along the direction of motion and the absence of vertical changes impair the homogeneity of the body. Relative inhomogeneity reduces Boltzmann entropy.

On the other hand, the separation of the concept of heat from kinetic energy will leave out the change in the numerator (heat) of the Clausius fraction and, due to the increase in the denominator (temperature), its (entropy) will decrease. Entropy thus decreases in proportion to the relative slowdown of the flow of time (redshift). Then the body remains in its inertial state of motion as it will not spontaneously transit to a state of lower entropy.

By stopping abruptly, at the moment of collision with the obstacle, the body temperature is just like before the collision, the kinetic energy goes into heat and the entropy of the body increases. The glass in flight only when it hits an obstacle breaks down in accordance with the increase in clutter due to the increase in entropy (in both situations: the glass flies and breaks the body, the glass is stationary and the body hits it).

`http://izvor.ba/`
November 1, 2019.

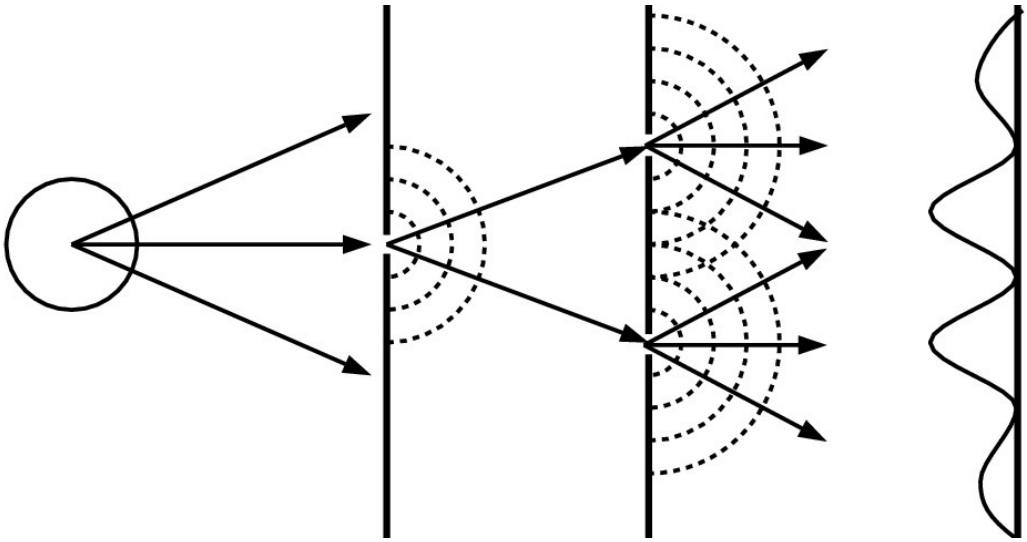
1.17 Double slit

Can the theory of information say something about the famous experiment *double-slit* of quantum mechanics, a friend asks me and asks me to adapt the matter a little to non-mathematicians. Of course, I answer and continue: if you view the vacuum as one big old ocean of uncertainty, and uncertainty as the kind of information, then the story flows on its own.

First of all, explain to the acquaintances that the "double slit" was *Young*[39] an experiment of 1801 in which he proved the interference of light and that light was not corpuscular as it was described by *Newton* in his "Optics" (1704). Only quantum mechanics, after the *Louis de Broglie* (1924) hypothesis of the wave nature of all matter, showed that they were both right. *Davisson*[40] and *Germer*[41] (1927) found the similar particle-wave duality with electrons, and then the duality was confirmed in atoms and molecules.

The device consists of a source of particle-wave beams perpendicular to two parallel straight obstacles, the first with two close and narrow vertical slits (openings) and the second is a detector, a screen for measuring the total energy flow of the beam. When the two slits are open, the beam passes through both, interfering, and dark and light streaks with more and less arriving energy, characteristic of wave diffraction, appear on the screen. When one slit is closed, there is no interference between the plates and no diffraction at the end. A picture of the hits is displayed, as in the case of a part of a jet of balls that would pass through the opening.

[39] Thomas Young (1773-1829), British physicist.
[40] Clinton Davisson (1881-1958), American physicist.
[41] Lester Germer (1896-1971), American physicist.



Slika 1.1: Interference.

This dual particle-wave nature of matter is already mysterious, but with this experiment there is something even more strangely about it. When beam particles are released from the source one by one, temporally separated at longer or shorter intervals in a random or other way, the screen image is repeated: passing through only one slit the particles do not have diffraction, when they go through both they have it.

The math is clear here (to connoisseurs). We summarize the vectors of the abstract Hilbert space that now represent quantum states of passage through slits, and interpret Born rule for probabilities. One part of (beam) particles bounces off the first obstacle, the second part passes and reaches the second obstacle (detector), we calculate the probability density and find that the measurement confirms exactly what was calculated.

Quantum mechanics is, in this sense, an incredibly accurate theory, and yet it is inexplicably intuitively mysterious. Because of the latter, it may be just so tolerant that the interference of a single particle with itself as it passes through two openings is understood as its splitting before and joining after the two-slits obstacle. This "possibility" of a particle-wave probability is for now the most convincing thing quantum mechanics has in this case.

Everett, who (1957) proposed the idea of *many worlds* of quantum mechanics in this fission, saw the appearance of the same particle from different pseudo-realities and their interference. The alternative possibility is again in the most probable nature, now information. A *particle* traveling through a *vacuum* (e.g. a photon) would acquire information along the way, it would communicate with the "void" and create its past, and as the particle does not grow as a result, the vacuum grows.

Both phenomena are possible, that in the quantum world coincidences are so significant that a particle can appear in several places at once and, on the other hand, that empty space is a kind of uncertainty, i.e. information, to have the potential of communication, particle-waves and interactions. Then the size of the space speaks of its past and that past can affect the present in other ways as well. The mentioned method is the interference of past and present corresponding wave-particles.

The law of quantity conservation limits the data volume of a given system, not their internal changes. When both slits are open the path of the particle is not well-defined, there are still choices and the particle is dominated by a vacuum ripple, a communication that is

a cyclical exchange of uncertainties. When one slit is closed, there is no uncertainty and the particle and its path are declared. The information is then formed and there is no room for the virtual influence of the vacuum. Computing and the screen image show the corpuscular behavior of the emission.

This description, a space that remembers and acts, is just as sufficient to explain the paradox of the double opening as the first (Everett's). As with rolling the dice (six numbers), uncertainty is always limited by options that can only be realized in information. The vacuum communicates with everything that passes through it and, therefore, has a huge potential of information. It is therefore a history of passages, a reservoir of possibilities that is constantly growing, because it always receives news from its passengers. Virtual vacuum waves are interference from those pasts, like waves on the surface of the ocean under which there is a lot of water. As geological deposits, or ourselves, after all, who we are and what we have experienced or inherited, *space remembers*.

Outside the temporal nature of mathematics will also support this second explanation of the "double slit" experiment. She can add it as a kind of generalization of the *Mach principle* that Einstein once advocated by reconciling the relativity of motion and the spillage of water from the basin as the vessel and liquid rotate. All the matter of the universe creates one gravitational field within which bodies can rotate, and now we only add that that field also contains the history of the universe.

Such an explanation is encouraged by the very nature of the *interference* of waves whose name is, I will note, inconsistent with the conception of the independence of random events of probability theory. At the root of the word "interfere" is an action on something, an addiction, dependence, while wave interference actually expresses their independence. Particles that attract or repel are dependent, to say not as the colors of light that we can always put together in white, and again (Newton's prism) decompose them into intact ingredients. White is dependent on its components, but its components are not interconnected.

If two random events are independent, then they are not mutually exclusive, and if they are excluded then they are dependent. This is from the probability theory[42], which, because of the form of quantum states, implies the above explanation. However, it is only the beginning of a much more interesting story about information in physics.

`http://izvor.ba/`
November 8, 2019.

1.18 Piling of history

Space, time and matter are forms of information. They communicate because uncertainty is objective for them, they are compelled to do so, and because they do not have everything they need. Another important feature of information of interest to today's topic is its uniqueness.

Before we throw a coin, dice or pull a ball out of a lotto drum we have an uncertainty whose quantity increases with the number of options. What are further important to us in such random events are quantities – that before and after realization we receive exactly the same amount of uncertainty. This is the meaning of the law of conservation information and the reason for calling both of these states by the same name. Information is a particularly defined measure of data that remains constant as data (of the closed physical system) change.

[42] see the proof in my book "Quantum Mechanics" [4]

There is more than one data in the uncertainty of one random event, but the outcome is only one. In addition, there will be no outcome of something that was not in the previous possibilities. When we roll a dice and each of the numbers, from one to six, has an equal chance that means that the number seven has no chance. Let's add this to the previous one as (some new) law of conservation the data itself.

Two types of uncertainty, before and after a random event, or two types of information, are called potential and current information to distinguish it. As well as generally well-chosen names in mathematics, these should be useful.

The names deliberately allude to potential and kinetic energy whose difference (kinetic minus potential) is *Lagrangian*. The energy output for a given time is called *action*, so the equivalents to the information mentioned are now potential and actual (kinetic) effect. The principle of least action (the least change of energy) thus becomes the principle of least information, which is again in line with the previous one, noting that such situations are not yet considered in physics.

This is how space looks from the point of view of information theory, it communicates with passing bodies, inevitably buying something from their history. It goes without saying that an elementary particle does not grow along with the growth of its past, which means that it leaves all the excess news with a vacuum (and other bodies if appropriate). The result is a constant accumulation of the history of the world in physical bodies and in space itself.

That the structure of a body, such as *geological deposits* or radioactive carbon, can sometimes give us information about itself is not some discovery, but it is also space itself. Modern physics knows that *vacuum* has its dynamics, which it is full of virtual particles, and this is also not disputed. The originality is that the vacuum is *history store*.

Certainty has no information and repeated news is no longer informative. Repeating the same physical information is not in the interaction options. We do not communicate with literal replicas and that is why we have so many differences in the perception of the world. That is why space is growing (by the amount of uncertainty) and is never the same, and its changes offer us a hidden history.

As for the "void", there are two major consequences of this principled uniqueness of information that it now makes sense to promote. The first is its expansion, and the second is the effect of the past of the universe on the present. Space is made up of potential information, and especially because it can be realized, and concealment and unexpectedness are its essence, as well as the action itself.

Let's take a look at how the new explanation works with the paradox of the principle of minimalism that I cited earlier as a conflict of the laws of maintaining information and evolution towards a possibly less complex future. The same has been observed in the literature as an entropy paradox because of the reversibility of time in quantum mechanics.

Let us now add the following reasons: the overall information of the universe is constant. The eventual lack of information in a substance resulting from the principle of minimalism (it evolves into less informative) is always equal to the resulting excess of the history of the universe contained in space.

The discoveries of 20th century astronomy can confirm something more. In 1912, the American astronomer Vesto *Slipher*[43] discovered the *redshift* of distant galaxies, which was later interpreted as moving galaxies away from Earth. Consistently, we add to the (new) informatics that the temperature of these galaxies (of the same state of matter) is higher,

[43]Vesto Slipher, 1875-1969

that the heat is not the same as the kinetic energy (not increased by departure), so we get a smaller number for the Clausius *entropy* (quotient of heat and temperature). Since light from distant galaxies has traveled a long way, entropy in the past is less and this is consistent with the second law of thermodynamics – that entropy increases.

Russian mathematician and physicist *Friedmann* [44] in 1922 derived from Einstein's field equations theoretical proof of the expansion of the universe. Independently of it, a Belgian Catholic priest and professor of astronomy and physics *Lemaître*[45], came to similar conclusions in 1927, and American astronomer *Hubble*[46] confirmed his theoretical results two years later with telescope observations.

According to the cosmological principle, the observation that the spatial distribution of the matter of the universe is homogeneous and isotropic, viewed on a large enough scale, concluded that the galaxies are moving away from each other. Roughly speaking, they are like stains on a balloon we inflate. This conclusion is consistent with the above findings if we accept that the accumulation of history in space is manifested by the expansion of the universe.

Moreover, from a constant increase per unit volume, we can also deduce the acceleration of this expansion of the universe, and thus be included in the latest discoveries in astronomy and theorizing about *dark energy*. Within the same hypotheses would be the consideration of *dark matter* as the gravitational action of the aforementioned past to the present if comparisons of the movements of galaxies today and the concentration of their matter would meet some expectations.

But, from the point of view of diversity of information, it would seem strange that this is all that lies in "dark energy" and "dark matter", even if part of this theory is fact.

`http://izvor.ba/`
November 15, 2019

1.19 Classical force

The concept of classical force in today's physics is in crisis. This crisis began with the theory of relativity and was still only developing, but it jet did not reach the level of the former (al) chemistry of the popular *phlogiston*, which was eventually put to rest by *Lavoisier*[47] and *Priestley*[48] through their research and discovery of oxygen.

That is why I do not base the information on the determinants of classical force, because every time I tried it something wasn't good – I say in answer to the reader's question.

First of all, the definitions of the *force* that give acceleration to the mass and those that change the momentum give different values over time in the special theory of relativity, which were the same in classical physics. This is because the relative time, mass, and momentum of the body in motion relative to the observer at rest are so transformed that the two determinants of force do not agree.

I will explain this briefly without many formulas, except you should know that by increasing the velocity of the *inertial system* the relative units of time elongate, mass and energy grow, and also the momentum in the direction of motion, all in proportion to a coefficient

[44] Alexander Friedmann, 1888-1925
[45] Georges Lemaître, 1894-1966
[46] Edwin Hubble, 1889-1953
[47] Antoine Lavoisier (1743 -1794), French chemist.
[48] Joseph Priestley (1733-1804), English chemist.

called Lorentz and denoted by the Greek γ, the letter *gamma*. It is a number that grows from one to infinite when the speed rises from zero to speed of light. Inversely proportional to the coefficient the relative lengths are shortened.

Acceleration is the distance traveled in unit time by time, so acceleration in the direction of relative motion decreases in proportion to the third degree of the gamma (inversely proportional to the cube of that coefficient). Mass increases, so the force as a product of mass and acceleration (in the direction of motion) decreases with the square of the gamma. The calculations are longer, and this is just a sketch of the results.

In other words, the proper (own) observer of the moving system, which is stationary in that system and would notice uniform acceleration of its system with a constant thrust force, would be seen from the relative as he is losing of the force and even more losing on the acceleration. The acceleration drop is such that he could never reach the speed of light. Conversely, if it is externally seen his constant acceleration, the force and acceleration felt by the proper observer would have to grow to infinity.

The relative momentum of the body in the direction of motion grows in the same proportion as the elongation of units of time (slowing down of time), so the change of momentum in the unit of time remains the same for both, the relative and the proper observers. This means that the relative change of force, as the change of momentum over a given time, conflicts with the previous definition (force that accelerates mass). When you look for Lorenz's transformation of force, a special theory of relativity, in physics textbooks, you come across this term – that the relative force in the direction of motion equals the proper.

Interestingly, the lateral force by both definitions gives the same result. Namely, there is no relativistic contraction of lengths perpendicular to the direction of motion, so the calculation gives a decrease in the relative force proportional to the (reciprocal value) of the gamma coefficient. This is in line with the paradox of two trains I wrote about explaining Bernoulli's equation (section 1.10) according to which fluid moves the surrounding matter with motion. All the same, partial agreement is a disagreement.

The exact sciences, and especially mathematics, believe only those truths from which infinitely deductible deductions can be made, with always exactly the same correct consequences, and none of the classical definitions of force fulfills this criterion. Unlike classical force, for now, the principle of least action and the corresponding principle of information work well in all circumstances, which is why I exploit them so much.

According to the principles, the most probable events are most often realized, whatever the circumstances, so force should be defined as something that defies spontaneity. Force, then, is something that changes the likelihood, that in new circumstances the most common realizations would change their choices, that is, something that transforms greater information into lesser. In game theory, they would say that force turns information into misinformation, and then beyond that – to make a weaker chance of winning better.

These are sufficient reasons why the classical concept of force should be changed by the principle of least action, that is, information, and so I do – I add in the end to the correspondence mentioned.

`http://izvor.ba/`
November 22, 2019.

1.20 Fourier deriving

One of the greatest French mathematicians, Joseph *Fourier* (1768-1830), was the son of a tailor and an orphan of eight years. He began his education at the monastery, then at the military school, and reached to be a student at the French High School with the famous professors Laplace and Lagrange to obtain the chair of mathematics at the Polytechnic School in 1797.

Fourier was a participant in the French Revolution, a follower of Napoleon in Egypt (1798), a senior diplomat, secretary of the Egyptian Institute, since 1817 a member of the Académie des sciences, in 1826 a member of the French Academy (Académie française) interesting for the discovery of the after him named series.

He observed that trigonometric functions and sinusoids in particular behave analogously to waves. They are summed up accurately by simulating wave interference, and with enough corresponding sines in the sum they can mimic almost any analytical function. This universal substitution by the sum of *sine functions* may have given Louis de Broglie idea much later that in 1924 to come up with the hypothesis of *waves of matter*, but even if it is not the significance of Fourier analysis for quantum mechanics would prove enormous.

It is known that we can properly stretch the wire so that it is excited by the *standing waves*, stilling at some points, the nodes, between which it oscillates. Fourier mathematics collects and translates such waves as desired, forms pre-desired shapes and imitates every kind of trajectory of physical particles. His theorems prove that in mathematics there are no obstacles to the assumption of the wave structure of matter.

Establishing *bijections* (mutually one-to-one mappings) between matter and action, interaction and communication, we find that Fourier's development functions into sine series are equally valid to the (physical) information. Each vibration has a period, the reciprocal of it is *frequency*, and this can be associated with *energy*, momentum, and action. We note that the titration is both a data and a material phenomenon, and a deeper reason that the Schrödinger equation (1926) mimics information as well as material quantum phenomena, so let's return to the past.

Fourier's discovery was later repeatedly refined and generalized to others, today we know of any fragments of arbitrary function to make "any interference" with almost any desired shape. In other words, mathematics allows elementary bits of particle trajectories of all shapes to be formless. This is a guarantee of the non-contradictory nature of today's microcosm-based microcosm-uncertainties of the quantum physics. And uncertainty indicates the informational origin of matter.

Moreover, both wave functions and individual "freedoms", the items of information of perception, which is a scalar product of the vector of intelligence and hierarchy, can be properly regarded as complex numbers. Therefore, it is possible to treat the logarithms of the exponents of these complex numbers as good surrogates of physical information and then obtain periodicity similar to particles (logarithms of complex numbers are periodic functions). Again it is mathematics that allows such views; it proves their universality and contradiction. However, with these familiar views it is not the end of the use of the Fourier method.

Allocation of energy to frequency, and it to information and vice versa, and the establishment of equivalences among other physical quantities with information, for example by waves. It is a detail from the general abstract connections between truth and matter, from the known to the physics as yet undiscovered. They should be distinguished from the hypothetical duality of matter and force, the idea of the *supersymmetry* theory in which is

still the possibility that force and matter are not equally well-founded physical concepts.

The more familiar duality is the already elaborated, but still intuitively incomprehensible, physical resemblance of vectors and quantum mechanics operators. It leads to the duality of quantum states and processes with some of the strangest phenomena of theoretical physics that have been extensively vetted and used during the second half of the 20th century, but which are difficult to talk about.

Simply put, the basic concepts of quantum mechanics such as energy, momentum, or position, in addition to their classical physical values, which are expressed in techniques well known to systems of units based on pounds, meters, and seconds, have their dual forms in differential and other operators of mathematical analysis. These operators formally behave as physical quantities themselves, although they represent their evolutions.

To experts in mathematical analysis, this duality between the operator and the magnitude in quantum mechanics is a "normal"occurrence, but even the best routinely working missed to promote the development of research by Joseph Fourier, the French mathematician named after the University of Grenoble. It is today a major scientific center, especially in the fields of physics, information science and applied mathematics.

`http://izvor.ba/`
November 29, 2019

1.21 Law of large numbers

The *law of large numbers* of probability theory is another confirmation of the principle of information minimalism. When we want more and get less, it said. Here are the basic ones.

The Italian mathematician *Cardano* (Gerolamo Cardano, 1501–1576) stated that the accuracy of the statistical findings improved with the number of attempts. However, the Law of Large Numbers (LLN) was first proved by the Swiss mathematician *Bernoulli* (Jacob Bernoulli: Ars Conjectandi, 1713) for binary random variables and named it *golden theorem*, later named after him. The French mathematician Simeon *Poisson*[49] described this finding in detail in a book of the same name (La loi des grands nombres, 1837), followed by considerable research into the subject by *Chebyshev*[50], *Markov*[51], *Borel*[52], *Cantelli*[53], *Kolmogorov*[54] and *Khinchin*[55].

When we throw a fair coin, the chances of a tail or head falling are exactly half-half, but that does not mean that every ten throws will drop exactly five times the tails and five times the heads. In addition to the mathematical *expectation*, the mean, the estimate of the average deviation from the expected outcome is also important. This scatter around the expected value is usually measured by the so-called *dispersion*. Expectation and dispersion define mainly all of the interesting properties of the probability distribution of a random event, at least as far as mathematics is concerned.

[49] Siméon Denis Poisson 1781-1840, French mathematician, engineer and physicist.
[50] Pafnuty Chebyshev 1821-1894, Russian mathematician.
[51] Andrey Markov 1856-1922, Russian mathematician.
[52] Émile Borel 1871-1956, French mathematician.
[53] Francesco Paolo Cantelli 1875-1966, Italian mathematician.
[54] Andrey Kolmogorov 1903-1987, Russian mathematician.
[55] Aleksandr Khinchin 1894-1959, Russian mathematician.

Let's imagine some random event, an experiment, a trial, like throwing (can unfair) dice. Each of the outcomes has its own probability, a real number from zero to one, so the sum of all into one means that surely something will happen. The chances of a particular outcome can also be expressed by the percentage of its occurrence in a long series of repetitions of a trial, or by a coefficient, a quotient of the number of a given outcomes and all repetitions of the trial. The sum of all percentages is one hundred, and the sum of all quotients is one.

The sum of all (realized) outcomes is exactly equal to the number of trials, as is the sum of their mathematical expectations. Therefore, for each outcome, the difference in the number of realizations and expectations divided by the total number of trials tends to zero as the number of trials increases. These differences, deviations from the mean, are limited by dispersions, so that with an increasing number of options, the cumulative scattering of differences increases more slowly than the total number of trials. In other words, when the number of trials increases, the sum of all differences divided by the number of trials tends to zero.

It is, in short, a sketch of the "exact in the nebulosity of coincidence" that brings us to the law of large numbers in mathematics, which is why this law is a theorem. This implies that trials are independent: after ten tosses of a coin and after the tails falls in all ten cases, again the probability of a tail in the next throw is the same as at the beginning. On the other hand, when properly understood and used, LLN enables long-term forecasts in the insurance business, in eliminating random side factors in medicine, in reducing errors by repeating measurements.

The most famous application of LLN within the theory of probability is to reduce the Bernoulli distribution to the *Gaussian bell*. The first is the binomial distribution that we get, for example, by tossing a coin a hundred times and counting down the tails. It is more likely to be 40 tails than 30, but any number of outcomes of a tail, from one hundred to zero, has a chance. The list of probabilities of these numbers is a distribution. We can test it by throwing all one hundred coins at once or repeatedly throwing one coin one hundred times.

The law of large numbers then says that the mean values of outcomes by multiple repetitions of one hundred throws are increasingly grouped around (precisely defined) probabilities, speaking even of steps, the degree of that approximation. When, along the horizontal axis (abscissa) we place the markings of event numbers, tails in a hundred coin tosses, and their heights (ordinates) mean numbers of all realizations, we get a bell-shaped graph, the Gaussian probability distribution.

Strangely enough, but no stranger to the fact it is almost impossible for us to dictate a series of "random" numbers that would pass the randomness test. Randomness tests exist and are based on LLN, simply said, on calculated expectations and mean deviations from expectations. It's not easy to control by heart, so guessing the numbers around the Gaussian bell is probably at least a little, but statistically significant, missed from the curve shape of the graph. Intuition falls on those tests as early as guessing the relationship of, say, 40 tails per hundred throwing a coin against 30 tails per hundred, not to mention the amount of deviation of expectation in a thousand repetitions per hundred throws.

Our problem with random tests is that they are a matter of mathematics, and intuition is naive for such precision. This is why we use random tests to control gambling fraud. They respond to almost all "wise" attempts to mimic "natural coincidence" thanks to our underestimation of the law of chance or the inability to defeat it. By reducing the binomial distribution in the crowd to bell-shaped, they are the consequence of a decrease in uncertainty by increasing the volume of experiments, and this is again part of the general principle

of skimping emissions of actual information from potential.

To show that there are similar topics I demonstrate by a contradiction. By collecting various uncertainties to increase the possibility of an outcome, a creature might want to have all the potential outcomes. Desiring maximum freedom of action, precisely because of the law of large numbers, it would then be maximally deterministically driven and completely illiberal.

`http://izvor.ba/`
December 6, 2019.

1.22 Quantum calculus

The classic computer is based on mathematical logic, information theory, and electrical current technologies.

From the ancient Greeks we know of absolutely correct statements, and lately we are able to "calculate" them. Then there is the discovery that "true" and "false" can be replaced by binary digits – 1 and 0 – bits of information, and then with technical solutions "electricity is flowing" and "no electricity is flowing". Thus, microchips have become the brains of computers made of densely integrated circuits that simulate the operations of algebra logic.

Along with these developments, quantum physics also evolved during the 20th century. It is also based on mathematics, but vector algebra. Vectors are quantum states, that is, superposition of possibilities, which we can call probability distributions. The phenomena of physics to which the laws of conservation apply are symmetric (Emmy Noether theorem) and quantified (because finite sets cannot be their proper subsets unlike infinite ones), and so are physical information. That is why information can travel almost without loss and is always discreet.

From the point of view of stinginess in principle, the losses will be even smaller if we transmit them in the form of uncertainty, as unexplained outcomes that dominate the microcosm. These are the vectors, the arrays that define the probabilities of realizing certain possibilities of quantum states, which are always particles because of the finite divisibility of information.

From mathematical views, the proofs, to legal paragraphs and political statements, all forms of communication are in portions, in steps, and such are the processing of information in computers, classical and quantum. On the other hand, because they are distributions, the vectors of quantum states are unit norms (lengths), and then so are the operators that map them (unitary).

The symmetry of quantum processes means some stability, the change in state similar (inherent, in eigenvector), but also reversibility, a feature that does not lose the previous information by copying. The most famous quantum operators are the Hadamard and Pauli matrices, and the search for their concrete representations becomes the daily work of many physicists in the world.

So, instead of classic electric circuits for the flow of individual bits, the quantum processor contains quantum gates. They usually map two-component vectors, the qubits of the information, and because physical information is always the true thing (it can't happen that could be proved it can't happen) and because of preferring uncertainty over declaring, it's a more natural way. Another story is the energy consumption.

Changing uncertainty into certainty, the emission of active information from the passive is some action (the product of a change in energy and elapsed time). Hence, I derive the

principle of least action known in physics, which now stems from the principle of minimalism (information), which again comes from the knowledge that more likely events are less informative and therefore more frequent.

In 1961 Landauer noticed almost the same. He emphasized that the cancellation of information is a wasteful process, because deleting a record in a molecule at some thermodynamic position will change the entropy of the schedule. If the process takes place at a given temperature, the product of temperature and entropy change is work, so Landauer concluded that to change the information someone has to pay an energy bill.

We can see from the following two examples that the energy calculus of quantum computers can be far more expensive than classic computers.

Let's imagine for us a "black box" that transforms each of two signals, 1 or 0, into one of those two signals. A classic computer will find out what the "box" does with two passes, separately mapping by "box" the zero then one. However, one copy is sufficient for a quantum, because it does not copy bit by bit, but qubit (both bits at a time). Well, there is no difference, you would say if there is no the next example.

To examine a more complex "black box" with a hundred input qubits that make up a hundred multiples of two bit variations, which is a decimal number written at about 30 digits, would take billions of years for a classic computer, even if it took only a millionth of a second for a pass. A quantum computer would do the job in a moment, in just a hundred passes.

Anything that a quantum computer can calculate cans the classic too, if it has enough time and memory. What used to be electronic lamps, transistors, and integral circuits was this formality, which is further modified by the representations of the Hadamard gate, Pauli matrices, and other recent from quantum physics theory and practice. We get the familiar but with daunting differences in capabilities, with fantastic benefits and new challenges.

The energy bill is as many times as the temperature at which the process takes place, so quantum computers need to be cooled to absolute zero. Low temperature in contrast to the environment enhances the effect of the principle of information and, therefore, reduces the noise of the environment, but cooling costs and the pan becomes more expensive when asked the cheaper pie.

There are also differences in the type of results. Quantum computing is no longer a clear deterministic process, it is a mapping of superposition of options, the probability distributions, so repetition will not produce the same results but similar even when the law of large numbers makes the quantum calculus very "exact" in those most complicated situations for which it is made.

`http://izvor.ba/`
December 13, 2019.

1.23 EPR paradox

After confirming the theories of relativity, in the time of the transition to theories with gradual advance of distance, *locality*, in 1935 the joint scientific work of Einstein, Podolsky and Rosen (see [21]) on the question of completeness of quantum-mechanical description of physical reality appeared.

This work is considered to be the discovery of *non-locality* in the process of wave packet reduction, initially only as the *EPR paradox*, but by the time and physics of *quantum entanglement* too.

The first two of only four pages of the EPR text discuss the impossibility of simultaneously determining the momentum and position of a particle. In the structure of quantum mechanics is Hilbert's abstract algebra. Quantum states (particles) are vectors, quantum states evolutions, interactions, and measurements are linear unitary operators. The multiplication of states by the operator is the change of state, and the multiplication of evolution by the operator is the change of evolution. Unlike ordinary numbers, these multiplications are not always commutative (they depend on the order) and this is the first problem.

That quantum mechanics is a representation of the aforementioned abstract algebra was noticed earlier – the non-commutativity of the operator was also known – but it was not until after *Heisenberg* and 1927 that physical meaning was sought in this. His proposition is *uncertainty relations* that have a classical algebraic and informatics sense.

In order to locate the electron as accurately as possible, light is used as short wavelength as possible. But such has higher energy and in the collision increases the unpredictability of the electron momentum. Using his "thought microscope" and his previous knowledge of physics, Heisenberg discovered that the product of the indefiniteness of the position and momentum of a particle is, at best, of the order of magnitude of the Planck constant, a quantum of action.

The meaning of the Heisenberg discovery in the algebra of linear noncommutative operators was quickly noticed. The difference between the action of the first and second operators and the second and the first on the quantum state of the order is the magnitude of the quantum of action, and this algebraic result is called the *uncertainty principle*.

Specifically, the position operator changes the position of the electron and sets values differently from those that would be created by changing the momentum (mass and velocity) by the action of the momentum operator precisely because the two operators are algebraically dependent. The action of one influence the action of the other and the importance of the order of application (multiplication) of the position and momentum operators becomes the dependence of the physical quantities interpreted.

When operators represent dependent events, the first changes the domain of the second, producing a different range of finite consequences than when the first distorts the domain of the first. The difference between the outcomes of these successive actions is, at best, a quantum of action, further adding we can say the least possible transfer of information. However, when we say that operators represent independent events, it means that they do not mutually usurp each other's domains, so it doesn't matter which one acts first.

Therefore, quantum entanglement, or reduction of the quantum properties, has an equivalent in the non-commutativity of the operators representing them. The following is the IT sense.

Dependent quantities are coupled and measurement is the interaction of particles and apparatus, their communication. The uncertainty relations tell us that by subtracting the position uncertainty, the particle momentum uncertainty increases, that is, the increase in the active information of one property is accompanied by a decrease in the other, with the products being constant. We understand this directly as the conservation (amount) of relevant information of the object itself but also of its "perception" by the measuring devices.

I will add, because of the law of conservation, we have a finite divisibility of (any) information, its appearance and perception in limited portions, so in order to increase say the resolution of the image in pixels in some restricted medium then we need to reduce the frame rate and make the motion unclear. I have written about it before, but not as here with an emphasis on the coherence, interdependence of phenomena, both as in the object itself so in its perception.

The following APR text on the next two pages discusses an analogous slightly more complex situation. Two systems (particles) that interact after which there is no communication between them are observed, but the first system (only the first one) is measured. The interaction is "wave packet reduction" or "quantum entanglement". Then we make measurements on the first system, and the algebra of quantum mechanics shows the changes of the second system as well!

The paradox comes from the timeless nature of formulas, because the other system can be very remote. Einstein called the effect "a phantom action at a distance" and to keep the absurd from being abstract, he came up with an example with gloves. A *pair of gloves* was placed in two separate boxes, very separate. Opening one and knowing that it is, say, the left one, at the same time we find out that in the far other box is the right glove before the light could come to inform us about it.

To overcome the EPR paradox, at one time, the most serious suggestion was to look for *hidden parameters*. Quantum mechanics is assumed by incomplete, the very principle of uncertainty is attacked (good God does not gamble – Einstein). It also struck at the very algebraic settings of quantum mechanics, if mathematics itself could not be doubted. The public was triggered by bombastic headlines in newspapers like "Einstein destroys quantum mechanics," but the whole thing was quickly forgotten. It was not considered scientifically serious.

Three decades later, the Irish physicist John Stewart Bell (1928-1990) (On the APR paradox, 1964, [20]) found a contradiction in the proposition of "hidden parameters": that the difficulties of this paradox can be resolved by variables added to quantum mechanics. They would restore quantum mechanics causality (randomness) and locality (gradual action), but his theorem proved the incompatibility of such an idea with the statistical character of quantum mechanics. The work went unnoticed.

The turning point in today's great and growing interest of physicists in quantum entanglement was the French Experiments (1976), a kind of new shock. Much to the surprise of confused authors who experimented to challenge Bell's theorem, they confirmed the "phantom" consequences of quantum reduction. They have opened a new chapter of physics, which is a special topic for us.

`http://izvor.ba/`
December 20, 2019.

1.24 Entropy generalization

Questions, suspicions and interpretations of "principled minimalism of information emission" continue to be addressed. Moreover, they seem to grow with the reinforcement of the theory.

Few today criticize the law of information conservation, and its "shyness" holds good too, even my explanation of thermodynamics, but some generalizations of entropy seem to be waiting for better times.

Let's say that *imperialization* and *feminization* of society are part of the broader aforementioned principle, along with speculation about the growth of the ability to absorb different nations with the former and decline with the latter. I'll explain.

The term *entropy* (Greek: turn inwards) was introduced as a term in 1865 by the German physicist and mathematician Rudolf Clausius, who considered it a measure of the energy of a closed system of a heat engine that could no longer be converted into work. For him it

was just an abbreviation of calculations, a quotient of heat (energy) and temperature. He found that in the circular process, the entropy increase was proportional to the heat loss and inversely proportional to the temperature.

In the *Shannon* definition (1948) the Clausius fraction became a measure of lost information. There are more spread and uniform than thick arrangements of balls in the boxes, and so the air molecules in the room expand. Entropy is the logarithm of the number of schedules, and probability is their reciprocal value. The logarithmic definition of entropy is the *Boltzmann's* of 1897, initially unaccepted because of the unrecognized idea of atoms or *molecules* of gas. Entropy thus means dispersal and a measure of disorder.

My considerations begin with these definitions. Set forth, I enclose to them the law of conservation of information and its minimalism, and this somewhat changes the essence. Theories are known to make sense of the facts, which here comes with the environment of a heat engine that stores overall information.

The oscillation of the room molecules is transferred to the colder outer walls. The *heat* and *temperature* go out, the oscillation weakens and the entropy increased. As entropy is a quotient of heat and temperature, the temperature decreases faster, so we conclude that the oscillation of molecules is more important to temperature than to heat. Both are spontaneously decreasing inside, and the transfer of heat from the body higher to the adjacent body of lower temperature (note: from less to greater entropy) is called the second law of thermodynamics. The first is the law of energy conservation.

Information goes away with heat, and it's okay to say that increasing entropy is proportional to losing information. This is also consistent with the interpretation of information by *action* (the product of energy and duration, or momentum and length). As the temperature decreases, the vibrancy of the molecule decreases, which we understand here in proportion to the ability to emit information. The alleged resulting *disorder* is proportional to the lack of heat, and what remains is an amorphous state of uniform distributions, of the most numerous combinations, and therefore most likely and least informative.

The principles of information are above the parts of the physics and laws of thermodynamics. In this broader concept, entropy is also explained by the law of *inertia*: the body does not spontaneously transit from the rest to motion, because the relative entropy in motion is less than its own (proper)!

Hence *gravity* decreases entropy (many disagree with this statement), so the body accelerates towards a stronger field to maintain entropy. In particular, falling off the glass from the table and breaking it is an irreversible process, because the entropy of the stopped glass increases (the molecules break apart). The information of the glass goes partly to the substrate and very small to the space itself. The actions are exchanged.

Conversely, if a glass at rest is hit by a moving object and broken, the object and the glass interact. There is no collision without changing the amount of movement, the momentum. The object slows down, and the pieces of the glass get speedily relaying information, the entropy of both bodies grows.

Information is not only a mere constituent of a substance, but also of life. The pursuit of inaction, inertia, is a form of principled shyness of information, self-directed, weak and persistent universal force. *A living being* with excess is information (and action) created by parsimonious, which is then diluted for the same principle. Loss of information is a loss of choice and action.

For quantum states, reducing individual options comes with entanglement (fidelity), for an individual in a society with denser regulations, but in any case, reducing information means greater causality and dependency, and decreasing risk and aggression. Psychologically

or economically, we see it as an increase in safety or efficiency.

In general, I call the processes with spontaneous growth of generalized entropy inherent in every form of living matter (as well as in inanimate) "feminizations", and opposite and slightly less frequent aspirations as "imperializations".

An empire is, by the classical political definition, a unit that brings together multiple peoples under one authority, and in the abstract it would be an information system with an excess of imposition in external uncertainties, risks, initiatives, aggression. Imperial and feminized societies come in two opposite directions of the "options quantity", outward and inward, as two formal, ubiquitous and dual, but somewhat asymmetrical phenomena.

Thus, the empire is aggressive towards the outside world, in its presence, the outsiders "spontaneously" divide. Feminized society, on the contrary, governs itself and favors internal hardship; by editing it strives for uniformity, the consideration of equalizing everything, and when that is not possible, sorting and dividing. Two processes, imperialization and feminization, such as the storm at sea and the appeasement of one another overpower but never give up.

From our side looking, the nature, in the conflict of its opposites looks as it does not know where to go, it would not be with the uncertainty, and without it it cannot. Indeterminacy is its essence, but it chooses more likely outcomes. Without unpredictability there is no need for communication, we would have everything from the beginning, but the differences are so great that there are too many of them to the nature itself.

`http://izvor.ba/`
December 27, 2019.

1.25 Entanglement

Quantum entanglement becomes the new "gold mine" of physics. It is a magnet for geniuses eager to prove themselves, for the prestige of the institute; the armies have their expectations, because it is more generous for discoveries than even tapping CERN on a "standard model" of particles.

Two physical phenomena are said to be coupled (entangled) when a change on one causes a change on the other. They may thus be altered by the law of conservation, the principle of least action, or some third necessity. The best known are the couplings with spin (internal angular momentum), but there are also Heisenberg uncertainties of momentum and position of one particle, as well as unknown virtual actions. My (hypo) thesis is that they are generalized by "ability" with "restriction" in "freedom" and the rest that follows from *perception information*.

Suppose we have a series of events where we can define pairs of sizes, called "abilities" and "constraints". The product of the respective couple (same event) is "freedom". The sum of all the "freedoms" of the physical system, its total freedom, is accompanied by information of perception. That part is formally undisputed and it would be hard to break it down in physics.

Vectors are basically arrays, in one case called "intelligence" and in the other "hierarchy". They are with the same number of members – *dimension of vector*, which in pairs define the same events – the first "ability" and the second "constraint".

However, in whatever dimensional coordinate system (observables) they are defined, two different vectors determine exactly one plane. They can always be reduced to a 2-dim

coordinate system, so information of perception has the two dimensions. It is one of the more recent views.

Quantum states are representations of vectors of Hilbert space, and their scalar products tell us about quantum entanglement. This can be generalized in part to the macro world. In the case of a simple physical substance, the scalar product is the total *freedom*. It also follows the principles of conservation, minimalism and the action of information, the most acceptable of which is the first. Coupled vectors are aligned; they are dependent random events and are monitored like electrons in an electromagnetic field, manufacturer and market, or grass and soil. And the occurrences of certainty are predictable even before the announcement.

For now, the typical "bound" situation of quantum mechanics is the emission of particles of the known *spin*. The spin is governed by the law of conservation and is easily registered by magnetic devices or polarization. If the total spin of the coupling is zero and two similar particles are emitted, they go in two opposite directions to maintain the total momentum, with the total spin zero. When the emissions are two electrons and the first spin is $+1/2$, then the spin of the second is $-1/2$. If these are two photons, the two spins are $+1$ and -1.

What can be weird here? When only one particle is registered, its positive and negative spin appears as random sizes. It is a matter checked by the measurements and laws of probability theory. But after registering a (random) first spin, spin of the second particle is no longer a random event, otherwise it would violate the law of conservation! However the second particle is distant from the first, and the time between the two measurements is short-lived, the spin of the second is always opposite to the first. It would be hard for experimental physics to notice this if the algebra of quantum mechanics did not impose it.

In the now famous, but on the time unnoticed, work on the APR paradox (see [20]), the Northern Irish physicist John *Bell* in 1964 considered the entanglements theoretically. I will not bother you with his calculations, which, incidentally, few understand today, but will just recount them. He assigned parameters to quantum states, either as a series of numbers or arrays of vectors, and found a contradiction. He proved that the assumption of missing variables in quantum mechanics algebra is inadmissible.

Knowing that the attempt to complete quantum mechanics to avoid the EPR paradox could not be reconciled with the stochastic nature of the micro world or with the previously tested the Born rule (probability of quantum measurements), Bell applied his analysis to other congregations with the same result. It was take or leave, all or nothing, and previous experiments had already confirmed the quantum theory extremely well. This unpleasant situation has long been ignored by physicists.

After more than a decade, the French were the first to experiment with Bell's findings and, as we know, the *entanglement paradox* became a physical reality. New physics has accepted experiments with instantaneous action at a distance and the absurd Bell's theorem that without coincidence there is no necessity and vice versa. To put it mildly, physics began with the belief that there was a conjunction, the interplay of the probabilistic nature of quantum mechanics and the law of conservation.

Further discoveries are additional surprises. When we act at one end of a pair of coupled quantum systems and thus achieve an "out of nowhere" appearance at the other far end, we produce a non-local effect similar to the transmission of information, that is, *quantum teleportation*. If we have digested this, then what about the alleged discovery of the "action of the present on the past", reported by the University of Vienna (2012)? This may be the *retrocausality* like the action of a virtual photon on a starting electron only after the action of the photon on another electron that I wrote about earlier has taken place.

If the past is an accumulation of information to the present, then retro causality (process

towards the past) could be related to causality (process towards the future). This is what the unitary operators (evolution) of quantum mechanics tell us, inevitable and in the program of perception information too.

`http://izvor.ba/`
January 3, 2020.

1.26 Dirichlet's principle

When a flock of pigeons arrives at the openings of the pigeon houses and if there are more pigeons than the openings, then at least two pigeons will arrive into at least one opening. This is the original version of the *Dirichlet*[56] principle[57], who used this idea in some problems in number theory, later called the *pigeonhole principle*.

When we have 11 objects arranged in ten boxes, then at least one box will contain at least two objects, or in each set of three natural numbers at least two have the same parity. From these obvious examples of application of the principle, the less obvious ones are quickly reached: in every group of 13 people there are two people born in the same month, in a group of 3,000 people at least nine celebrate their birthday on the same day, in a class with 35 students and 15 computers there is a computer with at least three students seated. In any gathering, with any number of persons, there are at least two persons with the same number of acquaintances.

Dirichlet's principle, of course, also applies in the same algebra whose representation is quantum mechanics. That's why the bombastic headlines are that quantum physics breaks this principle, in some popular scientific journals; in others it says that it bypasses it in its own way. Ming Cheng's recent experiment with nine other Chinese authors (see [5]) passes three photons through two polarizing filters, vertical and horizontal, avoiding the "pigeon principle".

When the filters detect the photon leakage, then there is no violation of the principles, but there are if the measurements are made only subsequently by diffraction of the photon at the output. The phenomenon is irresistibly reminiscent of the famous experiment *double-slit* when detecting on a slit before, eliminates diffraction, because that communication "consumes" information, and therefore energy. This is a reason not mentioned in the studies because for the information has not yet been officially acknowledged (not recognized) action.

For now, it is enough to know that in the micro-world appearances are easier transformed undefined. We write quantum states as superposition in probability distributions and say, for example, that a photon with a probability of 0.6 is vertically polarized and with a probability of 0.4 horizontally. The collapse of possibilities into one outcome is a measurement and continuation with and without collapse is not the same process, so maintaining flows spontaneously without aggressive interactions and declarations is opportunity to "defraud" the Dirichlet principle.

Yakir *Aharonov*, with five other authors, three years before the Chinese, published a similar study (see [11]). A summary of their work states: We find cases where three quantum particles are placed in two boxes without two particles being placed in the same box.

[56] Peter Gustav Lejeune Dirichlet (1805-1859), German mathematician.

[57] In mathematics, and particularly in potential theory, Dirichlet's principle is also the assumption that the minimizer of certain energy functional is a solution to Poisson's equation.

Furthermore, we show that the above *quantum pigeons principle* is only one of the related quantum effects, and we point to a very interesting structure of quantum mechanics that has gone unnoticed so far. Our result sheds new light on the very concepts of separability and correlation in quantum mechanics and on the nature of interactions. It also represents a new role for *entanglement*, complementary to the usual one.

They considered three particles and two boxes. I skip the record of the quantum state vector, but we understand that any two of the three particles can have some positive probability of finding in the same box. They further show that there are times when there is no chance of the two particles being together. They define orthogonal states, so there is a Hermitian operator whose eigenvalues (observables) are particles. The result is three particles (quantum states) in two boxes, but so there is at most one in each box!

Their analysis reveals significant differences in the observation of particles separately and together, further distinguishing between measurements in general (the quantum system on which more powerful interactions are performed) and non-measurements. The described differences, before and after separation, are always there as an integral part of quantum mechanics, they are attached, and each time we make a series of measurements we can divide the original set into several different, before and after selected subsets, according to the result of the final measurement and at each such we may notice a similar effect to the subset – that each division costs.

Finally, they conclude, general measurement is the measurement of a (unitary) operator with coupled (entangled) eigenvalues (observables), and it requires either particles in interaction or the consumption of some resources. The quantum effect of pigeons is, therefore, an example of a new aspect of quantum cohesion: measurement requires connectivity to produce correlations that otherwise exist in the immediate state.

I recall that the conjunction (entanglement) state of two quantum systems (particles) is represented by their scalar product. This product expresses the likelihood of interactions, compliance and dependence, so if the higher the greater the response of one condition to another, better monitoring without communication (emissions of information) and easier "under-the-radar" push-ups.

Said explanation is my informatics. We note that it clarifies the above descriptions even in the case of measurement differences on the sets of particles before and after separation. Better coupling results in reduced emission of information, such as connective tissue like electrons in an atom, in competitors in a *Nash equilibrium* (in which abandoning the initial strategy would endanger the player) or *living beings* (with more information than the physical substance possess) adapted to their environment. Behind all these phenomena is the universal tendency to reduce the emission of information.

Additional freedom is needed for an individual to leave the collective, or additional risk is needed for a player to exit a good position, analogous to the external energy required by an electron to leave the atom. This brings us back to the above idea that on the "double-slit" can pass more information or energy[58] anonymously, similar to the aforementioned photon effect of the Chinese.

`http://izvor.ba/`
January 10, 2020.

[58] thesis of my theory, otherwise unnoticed in physics

1.27 Cicle of Universe

I understood the uncertainty of information and its principled minimalism as well as the contribution of the information theory to Darwin's evolution, an anonymous reader tells me, and asks if I can somehow explain the *universe development* on the same way? The parts of the answers that I single out are speculative, but they are interesting and instructive.

General skimping by communication is a persistent and weak "force" of the universe. Hence comes the assumption that the young universe may have been (almost all) of the resultant, some highly unwanted information. Let's call it a substance. This idea is at first glance in stark contrast to the prevailing *big bang* theory today, but only at first glance.

At the beginning of the universe, there was minimally passive information, such as space and say tolerant bosons (force field carriers), but just as much action (product of momentum with length) as later. Due to the law of conservation action and the possibility of "melting" the effective information into potential, the space is grown. The losses of the first are gain for the second. This is consistent with the nature of the information and the assumption that *space remembers*. We would talk more physically about a fluid whose expansion is the *gravitational repulsion* of a substance, and about the *negative mass*.

There is no information without uncertainty. We communicate because we don't have everything, and because of that, repeated "news" is no longer news. Also, from moment to moment, the universe is re-emerging, mostly the most probable, which is in relation to the previous state, so the consequence of this kind of informatics is *chronological continuity* and consistency. This consistency is equal to the law of inertia, or the thesis that some (new) law of conservation also applies to probability, but we will not talk about them now.

If both the action and the momentum with the loss of the substance remain constant, then the phenomena as in the theory of relativity could occur. The relative mass and energy of the body increase, wavelengths increase (Doppler Effect), and units of length shorten and time slows down. Due to the slower flow of the present time, we see further galaxies (older ones) relatively faster, and the visible universe itself is getting bigger. On the other hand, older galaxies would have to move away relatively faster because of a higher substance that could then create more space.

We have come up with so many unusual allegations that they need to be reconsidered. That part of the general theory of relativity that arises by equating (curve) space with (tensor) energy could fit into this story. The special theory of relativity, for its side, has an additional part on the *Doppler effect* that would also fit here.

Namely, the relatively slower passage of the time of the light source observed in the movement, in the approach comes to us from our future so that at the moment of passing the two presents could be equalized. I now interpret the shortening of the relative wavelength of the incoming photons as less *smear* and higher probability of position. Conversely, the light of the outgoing source is still in the past, it defines less certain places with its larger wavelengths.

In short, the *future* is more likely. If we treat space and time formally so that the development of events is the displacement of the present in space-time, evolution towards the future becomes the "direction" to more probable events. I emphasize, because physics has no similar explanation for now.

So, the substance is less and time is getting slower. Gravity pulls where time slows down and a slow flow of time increases the relative inertia of the body, and thus *relative energy*. The same substances have increasing energies in such a way that the total energy *present* is constant, so the law of energy conservation is valid, as for the actions and information.

Consistent with information theory would be the dependence of the present on the past. We are what we were, and in part, things are so. More complex systems make it easier to remember their past in substance. They have fewer and fewer options for this if are simpler and for the simplest remain only space. The particles pile up the present and those without a trunk leave them along the way.

Photon traveling induces its electric field which induces magnetic and this again electric, from which we now see communication with space. The information is two-dimensional, with one axis always in the direction of motion, with the other alternately up (electro), then left (magnetic), down (electro) and right (magnetic), in four cycles with a total of 720 degrees of one period.

These are all (isometric) transformations of some rotation, and the double full rotation of photons in motion is described. It is *boson* (see [14]), but all *fermions* (see [15]) have similar rotation for two full angles, the particles on which the force fields (bosons) act. I will take an electron as an example of the trace of fermions in space.

Electrons communicate using photons. Each of them emits spheres of virtual photons in waves, which, if they interact with some other charge, become real. The probability of interaction decreases with the surface of the sphere, but the transmitted momentum remains constant. The action goes back in time, because the interaction is unpredictable and happens after the emission. At least two parts of the described process are new to physics: that the *Feynman* action with virtual photons is carried by the *sphere* and not by a line, then *retro causality*.

This story of the expansion of the universe may be a description of *dark energy*, and the story of the past a description of *dark matter*. The second would be pseudo-real (it acts more easily on us than we do on it), the disappearance of a substance would mean the formation of dark matter, the increase of saturation of space and the likelihood of popping virtual particles out of a vacuum, but these are topics for another occasion.

Information like "reality and truth" also offers greater miracles. Every part of the *exact theory* is not contradictory with any part of any other exact theory, so the happy-go-lucky theorizing of details, as long as it is in the field of truth, could be true, but that is also why no alternative "dark matter" would make us surprised.

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1.28 Hydrogen atom

The simplest physical system that contains potential interactions (not an isolated particle) is the *hydrogen atom*.

It is made up of one proton, one electron, and an electrostatic *Coulomb's potential*, attractive between the positive and negative charges of protons and electrons, which holds them together and decreases with the distance. As a quantum system, it is the only model of the atoms whose solution of the *Schrödinger equation* is, for now, quite known.

In short, the probabilities of finding electrons in a hydrogen atom are described by the product of three wave functions that define in turn its radius, meridian, and parallel (azimuth). Named somewhat freely, these are the three variables of the *spherical coordinate system*, respectively for distance from the nucleus of the atom (origin), the angle (phi) in the principal, horizontal plane, and the angle (theta) of deflection from that plane.

The radial function only has solutions for energies we know from *Bohr Model* of the Atom. An electron orbits around the nucleus like the planets of the solar system at quantum distances with an ordinal number that we call *principal quantum number*, n. The potential (negative) energy of an electron in orbit decreases with the square of that number, depending on the surface of the sphere and the number of possible standing waves.

From the point of view of information theory, we will say that negative energy has a negative effect (product of energy and time) and a deficit of information in relation to the neutral state. *Lack of information* is appealing. The electron then does not feel Coulomb's attractive force in stationary waves, and this is analogous to the weightless state in a satellite in a free fall in a gravitational field.

The (negative) energy must be added to the negative potential of the electron for it to escape from the nucleus to a higher orbit towards a neutral state of information. Unlike Bohr's model of atoms, the solutions of the Schrödinger equation can be positive energies, so *positive information*, then for free electrons.

Moving orbiting electrons around the nucleus produces the magnetization we find in the solution of the meridional factor (phi). It is associated only with integers that we call *magnetic quantum numbers*, m. The movement of electrons is an electric current that induces a magnetic field, so the presence of a magnetic quantum number testifies to the motion of electrons by *orbit*, although it is so smeared that it makes no sense to speak of more accurate positions.

The assumption of the exact positions of the electrons would lead the theory into contradiction. Therefore, we will say that due to the magnetic quantum number, the electron acts as the "outcome" of a random event, but also as a "possibility" due to smearing. These are two states of the process of shifting current and potential electron information. The electron not only oscillates in space (over time) around protons, but also dynamically. I'll explain.

The quantum system is a representation of Hilbert algebra, *quantum states* are its vectors, and quantum processes are the evolution of quantum states represented by linear unitary operators. In the end, state vectors are always some particles simply because "every property of information is discrete" (a consequence of the law of conservation), but as the mentioned operators themselves also make some vector space (dual to the first), then quantum processes are again "particles". Both types of vectors, each in its own way, are reduced to the same laws of quantum mechanics.

The third solution factor, which defines the azimuth (theta), depends on a nonnegative integer called the orbital, angular, or *azimuthal quantum number* of the ℓ label. Solving the Schrödinger equation, we find that it cannot be greater than the magnetic quantum number and that it is smaller than the principal quantum number.

The ordinal number of the shell, the principal quantum number is the upper (unattainable) value of the azimuthal quantum number, which is the (attainable) limit of the absolute values of the magnetic quantum number of electrons. In other words, the azimuthal quantum number, by its limitations, defines the degrees of freedom of the electrons in the atom, and thus the possibility of filling the shells of atoms.

The fourth quantum number is *spin* (s marks). The electron spin has only the positive or negative half value. It does not depend on the Coulomb force and does not come with the described solution of the Schrödinger equation. However, two electrons in an atom cannot have all four quantum numbers equal because this prohibits *Pauli exclusion principle*.

Starting from Pauli's principle and considering the above solutions, the electrons collapse into shells of atoms so that we obtain the *periodic table* of elements known in chemistry.

This is a kind of confirmation of quantum mechanics.

http://izvor.ba/
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Glava 2

Formalism

The following text was completed during May and June 2019 and printed as the second part of the book the year later. In the meantime, the first part was growing with the popular texts of my media column in "Izvor.ba". In these contributions, I will endeavor to address approximately the topics in this section of the book.

The basis of the story that follows is *information of perception*, previously defined (see [10]), which can be formally written as a scalar product of strings, and then vectors. It is bounded on the lower and upper values by the proportionality of the corresponding pairs, the factors of the summand mentioned, more precisely expressed by Theorem 2.1.3. Roughly speaking, the lower values of perception information belong to the *inanimate world* of physics, the upper to *living world*.

I also call information of perceptions the liberty, or total freedom, and its summands the individual freedoms, which will now come to light when we notice that these freedoms can be represented as Hartley information. We then look at the evidence that information of perception is two-dimensional, than that it is action, and that in the case of inanimate bodies of physics it gives the solution of the Euler-Lagrange equation, which's otherwise known proof, the derivation from the principle of least action, is enclosed here.

We treat the principle of least action here as the principle of least information, which means that the known trajectories of physics are now becoming a consequence of informatics. The formal evidence is there, and the intuitive explanations can mostly be found here in the first part the book. For example, recent (known) derivations of Einstein's equations from the least action are now becoming derivations of perception information, so there are several ways to reach them. The first is Einstein's, from the consideration of inertial motions and the relationship of energy-mass with geometry, the second is Riemann's with *geodesics* as lines of shortest distances, then lines of least energy change, and now lines of least emission of information.

The difficulties of the tensor calculus and the transformation into which such considerations introduce us are the main reasons for taking a break from mathematics, which is why I cut and delayed the continuation of the book for perhaps some later texts.

Finally, I cited a slight derivation of the Schrödinger equation that relates (my) information theory to quantum mechanics. In announcing the development of that text, which is not mentioned here, the principle of least action would be to form the Einstein equation of the field, to generalize that equation to an arbitrary number of D dimensions, and then to reduce it to the Klein–Gordon equation of quantum mechanics. However, it is a topic for another text.

2.1 Scalar product

Let $n \in \mathbb{N}$ is a given natural number. Vectors, arranged n-tuple of complex numbers:

$$\vec{I} = (I_1, I_2, \ldots, I_n), \quad \vec{H} = (H_1, H_2, \ldots, H_n), \tag{2.1}$$

where their components, that is, the coefficients $I_k, H_k \in \mathbb{C}$ for $k \in \{1, 2, \ldots, n\}$, we call respectively *inteligence* and *hierarchy*. Its *scalar product*, in popular quantum mechanic bra–ket notation, alias *Dirac*[1], we write

$$L = \langle \vec{I} | \vec{H} \rangle = I_1^* H_1 + I_2^* H_2 + \cdots + I_n^* H_n, \tag{2.2}$$

where I_k^* is the conjugate complex number of I_k and is called *information of perception*. When all the given numbers are real, scalar product which is also called the point product, we can write in a classic way $\langle \vec{I} | \vec{H} \rangle = \vec{I} \cdot \vec{H}$. Item $L_k = I_k^* H_k$ of information of perception we also call k-th *freedom*, and sum

$$L = \sum_{k=1}^{n} L_k = \sum_{k=1}^{n} I_k^* H_k \tag{2.3}$$

i.e. (2.2), we also call total freedom, or liberty.

Example 2.1.1 (Test)**.** *The person takes the test of the predicted various difficulty of the tasks on which he or she earns some points.*

Explanation. The given person takes the test with $n = 1, 2, 3, \ldots$ tasks. The solving the k-th task ($k = 1, 2, \ldots, n$) has $I_k \in \mathbb{R}$ points with the estimate that the k-th task has a weight of $H_k \in \mathbb{R}$. The intelligence and hierarchy coefficients here are real numbers and, of course, these numbers can be different from one another. More freedom L_k here corresponds to a "more interesting" task, one that requires greater mastery, and consequently I_k can be called the respondent *ability*, and H_k difficulty or *restriction*. □

When the freedoms L_k are integer, and to some extent real, the following expressions make sense to call some "quantity", number of options, or some mean (average) "probability" of those options:

$$N_k = e^{L_k}, \quad P_k = e^{-L_k}. \tag{2.4}$$

Here is $e = 2,71828\ldots$ *Euler*[2] number, base of natural logarithm. As $N_k P_k = 1$, so $\ln N_k = -\ln P_k$, and

$$L_k = \ln N_k \tag{2.5}$$

some respondent "Hartley information". This is further generalized to complex numbers of "capabilities" and "constraints" so that we can use the same terms in the domain of quantum mechanics.

Lemma 2.1.2. *From $a \geq b$ u $x \geq y$ it follows:*

$$ax + by \geq bx + ay, \tag{2.6}$$

where equality holds if and only if $a = b$ and $x = y$.

[1] Paul Dirac (1902-1984), English theoretical physicist.
[2] Leonhard Euler (1707-1783), Swiss-Russian mathematician.

Dokaz. The claim follows from:

$$(ax + by) - (bx + ay) = (ax - ay) - (bx - by) = (a - b)(x - y) \geq 0.$$

Because of the order relations, it is assumed that all these numbers are real. □

When the coefficients of intelligence and hierarchy are arranged both in descending order or both in ascending order, the liberty is maximized. When ordered in the opposite direction, one rising and the other decreasing, liberty is minimal. That this is true in the general case claim the following theorem [3].

Theorem 2.1.3. *Let the hierarchy coefficients of (2.1) be in descending order*

$$H_1 \geq H_2 \geq \cdots \geq H_n.$$

Let in ascending order ($I'_1 \leq I'_2 \leq \cdots \leq I'_n$) be the intelligence coefficients $\vec{I}'$, in decreasing order ($I''_1 \geq I''_2 \geq \cdots \geq I''_n$) intelligence coefficients $\vec{I}''$, and arbitrarily ordered coefficients (I_k) of intelligence $\vec{I}$ from (2.1). Then it is

$$L' \leq L \leq L'',$$

where $L' = \vec{I}' \cdot \vec{H}$ and $L'' = \vec{I}'' \cdot \vec{H}$ are responding liberties.

Dokaz. Let's start with an unordered series $\vec{I}$ and ordered it by *buble sort*[4], and every time a pair of adjacent coefficients is replaced, let us refer to the previous lemma.

We look at pairs of adjacent coefficients I_k and I_{k+1} for $k = 1, 2, \ldots, n - 1$ to sort in ascending order. If $I_k > I_{k+1}$ we replace their places in intelligence, and if not leave them where they are. Whenever a replacement occurs, according to the previous lemma, the value of liberty decreases. Passing through all pairs of adjacent coefficients is repeated over and over until there is at least one substitution in a given passage. The process is complete if there are no replacements and then $L' \leq L$.

Now look at pairs of adjacent coefficients I_k and I_{k+1} for $k = 1, 2, \ldots, n - 1$ to sort in descending order. If $I_k < I_{k+1}$ we replace the places in intelligence, and if not leave them where they are. Whenever a replacement occurs, according to the previous lemma, the value of the liberty increases. When the sorting process is complete, then $L \leq L''$.

This proves the theorem. □

When we have the sum of product pairs and the total liberty formed (2.2), that is

$$L = I_1H_1 + I_2H_2 + \cdots + I_nH_n, \tag{2.7}$$

then we can arrange the terms in an arbitrary manner, since the adding is commutative. Therefore, in short, we say that by multiplying larger ones with smaller (or smaller with larger) coefficients of intelligence and hierarchy, we get the minimum total freedom, and by multiplying larger ones with smaller (and smaller with smaller) coefficients of intelligence and hierarchy, we get maximum total freedom.

In the text below, we note that for a given natural number $n = 1, 2, 3, \ldots$ and n-tuples of the real numbers $\vec{u} = (u_1, u_2, \ldots, u_n)$ and $\vec{v} = (v_1, v_2, \ldots, v_n)$ there exists a n-dimensional Descartes rectangular *coordinate system* $OX_1X_2 \ldots X_n$ in which they are oriented lengths, that is, vectors, beginning in the origin of O and a vertex with a given set of coordinates.

[3] Example 1.5.3. from [10] and Theorem 1.2.7. from [8].

[4] Bubble sort: `https://en.wikipedia.org/wiki/Bubble_sort`

2.2 Vectors

Let n-dim is Descartes' rectangular coordinate system $OX_1X_2\dots X_n$ with real vectors $\vec{u} = (u_1, u_2, \dots, u_n)$ and $\vec{v} = (v_1, v_2, \dots, v_n)$. The *intensities* of these vectors are:

$$|\vec{u}| = \sqrt{u_1^2 + u_2^2 + \cdots + u_n^2}, \quad |\vec{v}| = \sqrt{v_1^2 + v_2^2 + \cdots + v_n^2}, \tag{2.8}$$

where the upper indices represent squares of real numbers. The orthogonal *projections* of these vectors on the coordinate axes are:

$$\vec{u}_{x_k} = |\vec{u}| \cos\alpha_k, \quad \vec{v}_{x_k} = |\vec{v}| \cos\beta_k, \quad k = 1, 2, \dots, n, \tag{2.9}$$

where α_k or β_k are the angles between the first or second vector and the X_k coordinate axis. Hence the following:

$$|\vec{u}|^2 = |\vec{u}|(\cos^2\alpha_1 + \cos^2\alpha_2 + \cdots + \cos^2\alpha_n), \tag{2.10}$$

$$|\vec{v}|^2 = |\vec{v}|(\cos^2\beta_1 + \cos^2\beta_2 + \cdots + \cos^2\beta_n), \tag{2.11}$$

that is

$$\cos^2\alpha_1 + \cos^2\alpha_2 + \cdots + \cos^2\alpha_n = 1, \tag{2.12}$$

when the vector is unit, $|\vec{u}| = 1$. I recall, in addition to another way, this explains the *Born*[5] law of probability of quantum states in the book Quantum Mechanics [4].

Let's take a closer look at that now. We only take the positions that are well known to us from elementary mathematics, and some prove then. First of all, these include decomposing vectors by coordinate axes; using orthogonal unit vectors of these axes, briefly say *orts*, then properties, performances of operations of addition and multiplication of the vectors so represented. This is part of the high school math lessons that we will repeat through examples.

Example 2.2.1 (Orts)**.** *Define the vectors given, $\vec{u}$ and $\vec{v}$, of $OX_1\dots X_n$ by using the orts and specify the properties of addition and multiplication.*

Solution. Denote the unit vectors of $OX_1\dots X_n$ by $\vec{e}_1, \dots, \vec{e}_n$, so that we can write

$$\vec{u} = u_1\vec{e}_1 + \cdots + u_n\vec{e}_n, \quad \vec{v} = v_1\vec{e}_1 + \cdots + v_n\vec{e}_n. \tag{2.13}$$

Adding the vectors is *commutative*, which means

$$\vec{u} + \vec{v} = \vec{v} + \vec{u}, \tag{2.14}$$

for each pair of vectors $\vec{u}$ and $\vec{v}$ and the property *linearity* gives

$$p\vec{u} + q\vec{v} = (pu_1 + qv_1)\vec{e}_1 + \cdots + (pu_n + qv_n)\vec{e}_n, \tag{2.15}$$

for any pair of real numbers p and q.

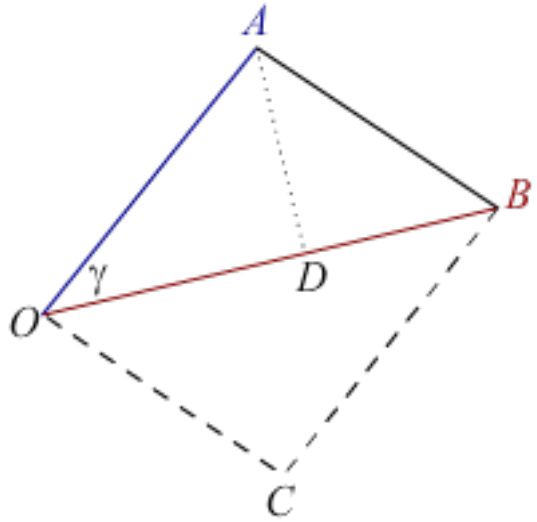
The scalar multiplication of vectors is also commutative. For scalar multiplication of given orts, the following applies:

$$\vec{e}_i \cdot \vec{e}_j = \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \tag{2.16}$$

where δ_{ij} is *Kronecker's*[6] *delta symbol*, for $i, j \in \{1, 2, \dots, n\}$, so for intensity we again find (2.8), as well as for the scalar product (2.2), which is easy to check. □

[5] Max Born (1882-1970), German-Jewish physicist and mathematician.
[6] Leopold Kronecker (1823-1891), Prussian mathematician.

As we know, vectors are equal if and only if they have the same direction, course and intensity. Therefore, we can *translate* (move in parallel) the given vectors so that $\vec{u} = \overrightarrow{OA}$ and $\vec{v} = \overrightarrow{OB}$, as seen in the figure 2.1.



Slika 2.1: Parallelogram.

The opposite sides of the $OABC$ parallelogram have equal vectors:

$$\vec{u} = \overrightarrow{OA} = \overrightarrow{CB}, \quad \vec{v} - \vec{u} = \overrightarrow{AB} = \overrightarrow{OC}, \tag{2.17}$$

and the angle between the diagonal OB of the parallelogram and the line OA is $\gamma = \angle(BOA)$. As we know from trigonometry, in an arbitrary triangle such as ΔOAB holds *cosine rule*

$$|AB|^2 = |OA|^2 + |OB|^2 - 2|OA||OB|\cos\gamma, \tag{2.18}$$

where $|AB|$ is length of side AB the given parallelogram, and the angle $\gamma = \angle(BOA)$.

Example 2.2.2 (Cosine rule)**.** *Prove the equation (2.18).*

Доказ. From right angle triangles ΔADB, ΔADO and *Pythagorean theorem* we get:

$$|AD|^2 = |AB|^2 - |BD|^2, \quad |AD|^2 = |AO|^2 - |OD|^2,$$

$$|AB|^2 - |BD|^2 = |AO|^2 - |OD|^2,$$

$$|AB|^2 = |AO|^2 + |BD|^2 - |OD|^2,$$

$$|AB|^2 = |AO|^2 + (|OB| - |OD|)^2 - |OD|^2,$$

$$|AB|^2 = |AO|^2 + (|OB|^2 + |OD|^2 - 2|OB||OD|) - |OD|^2,$$

$$|AB|^2 = |AO|^2 + |OB|^2 - 2|OB||OD|,$$

$$|AB|^2 = |OA|^2 + |OB|^2 - 2|OB||OA|\cos\gamma,$$

because $|OD| = |OA|\cos\gamma$. This is the required equality (2.18). □

When $\gamma = 90°$, then $\cos\gamma = 0$, ΔOAB is a right triangle with right angles to the vertex O, so (2.18) becomes Pythagoras' theorem. Therefore, the cosine rule generalizes Pythagoras' theorem to non-right triangles. Further, using the cosine rule, in addition to (2.2), one can derive another formula for the scalar product of the vectors.

Example 2.2.3 (Scalar product)**.** *To prove*

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\gamma, \tag{2.19}$$

using cosine rule.

Dokaz. According to the cosine rule and given marks it is

$$|\vec{w}|^2 = |\vec{u}|^2 + |\vec{v}|^2 - 2|\vec{u}||\vec{v}| \cos\gamma.$$

Then, by scalar product we find:

$$|\vec{w}|^2 = |\vec{v} - \vec{u}|^2 = (\vec{v} - \vec{u}) \cdot (\vec{v} - \vec{u}) =$$

$$= \vec{v} \cdot \vec{v} - \vec{u} \cdot \vec{v} - \vec{v} \cdot \vec{u} + \vec{u} \cdot \vec{u} = |\vec{u}|^2 + |\vec{v}|^2 - 2\vec{u} \cdot \vec{v}.$$

Comparing this with the cosine rule we obtain

$$\vec{u} \cdot \vec{v} = |\vec{u}||\vec{v}| \cos\gamma,$$

and that was what needed to be proven. □

Assuming we initially defined the scalar product with (2.2), we have

$$u_1 v_1 + u_2 v_2 + \cdots + u_n v_n = |\vec{u}||\vec{v}| \cos\gamma. \tag{2.20}$$

In the parallelogram 2.1, the vertical (orthogonal) *projection* of the point A on the diagonal OB is the point D. By the definition of cosine, again because of $|OD| = |OA| \cos\gamma$ (as in the proof of the cosine rule), we also have

$$\vec{u} \cdot \vec{v} = (|OA| \cos\gamma)|OB|. \tag{2.21}$$

In other words, the scalar product of two vectors is equal to the product of projection (length) of the first onto the second and the intensity of the second vector. And this is also known from elementary mathematics.

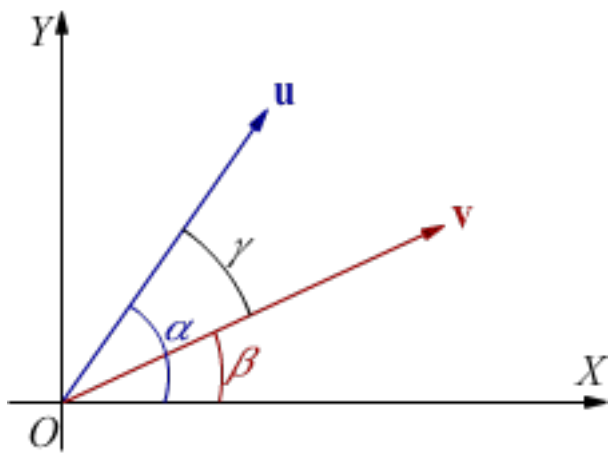
No matter how many coordinate axes a system has, the given vectors, if they are of different direction and common origin, define one and only one plane. In the figure 2.2 the vectors given are denoted by the bold, $\mathbf{u}$ and $\mathbf{v}$, in Descartes rectangular system OXY. The angles between the abscissa and these vectors are $\alpha = \angle(XO\mathbf{u})$ and $\beta = \angle(XO\mathbf{v})$, respectively, and the angle between the vectors is, of course, $\gamma = \angle(\mathbf{u}, \mathbf{v})$. Clearly it is

$$\gamma = \alpha - \beta. \tag{2.22}$$

Let us use this to prove the addition formula of the cosine of difference

$$\cos\gamma = \cos\alpha \cos\beta + \sin\alpha \sin\beta, \tag{2.23}$$

which is also a well-known trigonometry formula.



Slika 2.2: Single-plane vectors.

Example 2.2.4 (Cosine difference)**.** *Prove the addition formula (2.23).*

Dokaz. The *scalar product* of given vectors holds (2.20), so we have:

$$u_x v_x + u_y v_y = |\mathbf{u}||\mathbf{v}| \cos\gamma,$$

$$|\mathbf{u}| \cos\alpha |\mathbf{v}| \cos\beta + |\mathbf{u}| \sin\alpha |\mathbf{v}| \sin\beta = |\mathbf{u}||\mathbf{v}| \cos\gamma,$$

$$\cos\alpha \cos\beta + \sin\alpha \sin\beta = \cos\gamma,$$

which is the required formula (2.23). □

Note that *vector norms* (intensities) are now

$$|\mathbf{u}| = \sqrt{u_1^2 + u_2^2}, \quad |\mathbf{v}| = \sqrt{v_1^2 + v_2^2}, \tag{2.24}$$

and that this is a special case of (2.8). One particular case is the reduced form of formula (2.20) as used in the proof of the cosine of difference. However, the same "special case" of the reduction of perception information has far-reaching significance for us. If observable (physically measurable quantities) are representations of coordinate axes, then we can say that we reduced the perception information (L) from n observables (2.9) to only two "observables". Again, the theorem 2.1.3 holds for these two.

2.3 Action

We have just seen that the information of perception can always be reduced to a single plane. We know from earlier that uncertainty is the primary producer of information, that forces change uncertainty, and that both, the seizure of information from uncertainty and violence on it, are spontaneous events. The corresponding spontaneity is called the *principle of minimalism.*

Because of the principle of minimalism and the theorem 2.1.3, in the search for elementary physical information it is inevitable to consider Heisenberg's relations of uncertainty as a model[7]. The uncertainty of the momentum and position of the observed particle, both for

[7]see [8], Example 1.1.2.

each of the three spatial coordinates (length, width, and height), as well as the uncertainty of its energy and time, are good candidates for "ability" and "restriction" as the factors in the summands of "liberty" (2.2), that is, information of perception. Moreover, the nature of their products (when the first factor is greater the second is smaller, and vice versa) speaks of minimal "liberty".

For "Heisenberg Liberty", we define the scalar product

$$L = \Delta p_x \Delta x + \Delta p_y \Delta y + \Delta p_z \Delta z - \Delta E \Delta t, \tag{2.25}$$

where instead of items, like $\Delta p \Delta x \geq \frac{\hbar}{2}$ or $\Delta E \Delta t \geq \frac{\hbar}{2}$, the products can stand

$$\sigma_p \sigma_x \geq \frac{\hbar}{2}, \tag{2.26}$$

where σ is the usual designation for dispersion, here momentum and positions, followed by energy and time. As we know, $h = 6.626\,070\,15 \times 10^{-34}$ Js is *Planck constant*, and $\hbar = h/2\pi$ is its reduced form.

In the following theorem, we consider "perception information", uncertainty (2.25), in two inertial Descartes rectangular coordinate systems K and K' whereby K' moves along the abscissa at a constant speed v with respect to K, and at the initial moment $t = t' = 0$ coordinate axes coincide with them: $x = x'$, $y = y'$ and $z = z'$.

Theorem 2.3.1 (Lorentz transformation)**.** *Show that*

$$\begin{cases} \Delta p'_x = \gamma(\Delta p_x - \beta \Delta p_t) \\ \Delta p'_t = \gamma(\Delta p_t - \beta \Delta p_x), \end{cases} \qquad \begin{cases} \Delta x' = \gamma(\Delta x - \beta \Delta ct) \\ \Delta ct' = \gamma(\Delta ct - \beta \Delta x), \end{cases} \tag{2.27}$$

where $\gamma^2(1-\beta^2) = 1$. *With* $\beta = \frac{v}{c}$ *these become Lorentz transformations.*

Dokaz. The uncertainty (2.25) is the same in both systems K and K', so we have:

$$L' = \Delta p'_x \Delta x' - \Delta p'_t \Delta ct' =$$

$$= \gamma(\Delta p_x - \beta \Delta p_t) \cdot \gamma(\Delta x - \beta \Delta ct) - \gamma(\Delta p_t - \beta \Delta p_x) \cdot \gamma(\Delta ct - \beta \Delta x)$$
$$= \gamma^2(1-\beta^2)(\Delta p_x \Delta x - \Delta p_t \Delta ct) = L,$$

for $\Delta p_x \Delta ct = \Delta p_t \Delta x = 0$. From invariance, $L' \equiv L$, follows $\gamma^2(1-\beta^2) = 1$, which was to be shown. The interpretation *Lorentz*[8] of movement gives $\beta = \frac{v}{c}$. □

It is known that there are no changes perpendicular to the direction of motion (abscissa, i.e. x-axes), so transformations in the direction of ordinate and applicate (y and z axes) are ignored here. In the theorem, we give no special value to the relativistic gamma coefficient, $\gamma = (1-\beta^2)^{-1/2}$, nor do we use results other than the assumed *symmetry* in transforming the given uncertainties. We would do the same with dispersions (2.26). This makes it easier to interpret the coefficient γ not only in the special *theory of relativity*, as here, but still in the general too.

The use of only two dimensions to define transformations (2.27) is consistent with the prior said reducibility of perception information to 2-D. As is well known, the fourth Minkowski spacetime coordinate is often $x_4 = ict$, where $i^2 = -1$ for an imaginary unit and

[8] Hendrik Lorentz (1853-1928), Dutch physicist.

$c = 299\,792\,458$ m/s is *speed of light* in a vacuum. Then, for the fourth momentum coordinate is taken $p_t = iE/c$, hence the expression (2.25) and substitutions in the proof. The above theorem applies, of course, also to the transformations of the coordinates themselves, the direction of motion and time of the system K and K'. Due to the importance of this text, let us recall some historical findings[9].

Galileo[10] transformations

$$x' = x - v_x t, \quad y' = y - v_y t, \quad z' = z - v_z t, \quad t' = t, \tag{2.28}$$

were improved by the Lorentz's, primarily due to the experiments of Michaelson and Morley. Since 1887, M-M experiments have found that "light moves always at the same speed in a vacuum", about $c = 3 \times 10^8$ m/s, regardless of the speed of the source. The cognition of M-M experiments was taken by *Einstein* for the second principle in a paper he published in 1905, which is now known as the Special theory of relativity. His first principle was the relativity of motion, that "the physical laws of a system in which the observer is stationary do not depend on whether that system moves" uniformly straight linear to any other body or observer.

In Short, Einstein's spetial *principles of relativity* are 1: all uniformly straight linear motions are equal; and 2: the speed of light in a vacuum does not depend on the speed of the source.

Example 2.3.2. *Prove Lorentz transformations from the Einstein's principles.*

Solution. Einstein's relativity uses ct for the fourth coordinate, which is the length that light travels during time t. For system K' moving at velocity v with respect to system K along the abscissa, in the most general case we can put:

$$x' = \gamma(x - \beta\, ct), \quad y' = y, \quad z' = z, \quad ct' = A\, ct - Bx, \tag{2.29}$$

where γ, β, A, B are unknown numbers that have yet to be determined. The first and fourth coordinates are functions dependent on the mutual velocity v of movement of the two systems, and possibly only on the velocity of light c. These transformations would become Galilean's (2.28) if $\gamma = 1$, $\beta = v/c$, $A = 1$, and $B = 0$.

When an object is stationary in K' at position $x' = 0$ it moves at a constant velocity v by the abscissa of the system K, so that $x = vt$, so $\beta = v/c$ and $x' = \gamma(x - \frac{v}{c}\, ct)$. According to the principle of relativity, the inverse transformations between K' and K must have the same form but with the speed of the opposite sign, so we find $x = \gamma(x' + \frac{v}{c}\, ct')$ for the same coefficient γ. In this case $t = x/c$ whenever $t' = x'/c$ and by a change to the previous equations we then multiply by $xx' = \gamma^2(1 - v^2/c^2)xx'$. Hence, for the first equation:

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}, \quad \beta = \frac{v}{c}. \tag{2.30}$$

The first coefficient is also called the Lorentz factor and we have already mentioned it. The equation in (2.29) defining the transformation of time can be obtained from the conditions $x' = ct'$ and $x = ct$ by shifting to the previous spatial coordinates from which $ct' = \gamma(ct - \beta x)$. Therefore, the Lorentz transformations are:

$$x' = \gamma(x - \beta\, ct), \quad y' = y, \quad z' = z, \quad ct' = \gamma(ct - \beta x), \tag{2.31}$$

where γ and β are given with (2.30). □

[9] Taken from: [8], section 1.4.1. Lorentz transformations.

[10] Galileo Galilei (1564-1642), Italian mathematician.

The summands in (2.25), and then the sum of L, have a physical dimension of action. Therefore, the theorem 2.3.1 tells us that both information of perception and action are invariant to Lorentz transformations. An action is quantified, so it also applies to the *conservation law* analogous to information[11]. Therefore, we speculate that perception information and physical action are formally equivalent.

That this "guesswork" makes deeper sense is evident from the interpretation of *Pythagorean Theorem* as a kind of action. If we look at the length and duration of the environment of a given Minkowski space-time point (special relativity), we have a 4-interval

$$\Delta s^2 = \Delta x^2 + \Delta y^2 + \Delta z^2 - c^2 \Delta t^2, \tag{2.32}$$

where the above indices represent the squaring of the interval and the speed of light. The term generalizes Pythagoras' theorem to Minkowski's space-time, and is also invariant to transformations (2.31) as well as action (2.25).

Example 2.3.3. *Prove that (2.32) is an invariant of the Lorenz transformations.*

Solution. Using the marks provided, we have:

$$\Delta x' = \gamma(\Delta x - \beta\, c\Delta t), \quad c\Delta t' = \gamma(c\Delta t - \beta \Delta x),$$

$$(\Delta x')^2 = \gamma^2(\Delta x - \beta\, c\Delta t)^2, \quad c^2(\Delta t')^2 = \gamma^2(c\Delta t - \beta\Delta x)^2,$$

$$\begin{cases} (\Delta x')^2 = \gamma^2[(\Delta x)^2 - 2\beta\, \Delta x\, c\Delta t + \beta^2\, c^2(\Delta t)^2] \\ c^2(\Delta t')^2 = \gamma^2[c^2(\Delta t)^2 - 2\beta\, c\Delta t\, \Delta x + \beta^2\, (\Delta x)^2]. \end{cases}$$

Hence by subtraction:

$$\begin{aligned} (\Delta x')^2 - c^2(\Delta t')^2 &= \gamma^2[(\Delta x)^2 - c^2(\Delta t)^2] - \gamma^2\beta^2[(\Delta x)^2 - c^2(\Delta t)^2] \\ &= \gamma^2(1-\beta^2)[(\Delta x)^2 - c^2(\Delta t)^2] \\ &= (\Delta x)^2 - c^2(\Delta t)^2. \end{aligned}$$

Adding the square of the interval gives the other two coordinates $(\Delta s')^2 = (\Delta s)^2$, and that is what was to be proved. □

When (2.32) is multiplied by the mass m of the observed particle and divided by the time Δt changes in the position of the coordinate system, we obtain the expression (2.25). In other words, expression (2.32) multiplied by 4-action ($m\dot{s}$) will give expression (2.25), the previous form of action, that is, perception information. This is the basis for treating the 4-interval as action, or as information of perception.

If we look at the length and duration of the infinitesimal adjacencies of a given Minkowski space-time point, we have an infinitesimal 4-interval

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2, \tag{2.33}$$

where the above indices represent the squaring of the differentials. We also call these infinitesimal 4-intervals *metrics*. Same interval in spherical coordinates

$$\begin{cases} x = r\cos\varphi\sin\theta \\ y = r\sin\varphi\sin\theta \\ z = r\cos\theta, \end{cases} \qquad \begin{cases} r = \sqrt{x^2+y^2+z^2} \\ \varphi = \operatorname{arctg}\frac{y}{x} \\ \theta = \arccos\frac{z}{r}. \end{cases} \tag{2.34}$$

[11] The law of conservation information is about in my previous book, Physical Information, [2].

become

$$ds^2 = dr^2 + r^2 \sin^2\theta\, d\varphi^2 + r^2\, d\theta^2 - c^2 dt^2. \tag{2.35}$$

Further, in a centrally symmetric gravitational field this expression takes shape

$$ds^2 = \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2 \sin^2\theta\, d\varphi^2 + r^2\, d\theta^2 - \left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2, \tag{2.36}$$

where $G = 6,67 \times 10^{-11}$ N m^2/kg^2 is gravitational constant, M the mass of the body that produces gravity, and r the distance of the observed point from the center of gravity. This is the famous *Schwarzschild solution* of Einstein's equations. Also, we present this ds^2 as an action, and therefore as an information of perception.

In the presence of strong masses (large M), unlike *Newton*[12] gravity, Einstein's that can be derived from metric (2.36), in addition to draws the given body towards the center, squeezes it laterally. A man standing in a room in a very strong gravitational field would feel different force at the height of his head and legs. This would create the effect of stretching, thinning and tearing the decaying body. Therefore, it seems that from the Lorentz interval (2.35) it is impossible to obtain Schwarzschild's (2.36), or vice versa, but the following theorem stated it is not true.

We mean the spherical coordinate system $Or\varphi\theta$ with the center of gravity at the origin and look at the central symmetric field, so any two such systems will have the same angles φ and θ.

Theorem 2.3.4. *Differential transformations are given*

$$\begin{cases} dr = \chi\, dr' + i\gamma^{-1}\sqrt{1-\gamma^2\chi^2}\, cdt' \\ cdt = i\gamma\sqrt{1-\gamma^2\chi^2}\, dr' + \gamma^2\chi\, cdt', \end{cases} \tag{2.37}$$

where i is the imaginary unit ($i^2 = -1$), $\gamma = (1 - 2GM/rc^2)^{-1/2}$, and $\chi \in \mathbb{C}$ is an arbitrary parameter. Show that these are general transformations that translate the gravity metric (2.36) into an inertial metric (2.35).

Dokaz. We ignore coordinates that do not affect the result ($d\varphi = d\varphi'$ and $d\theta = d\theta'$) and start from the system:

$$\begin{cases} dr = \alpha_{rr}\, dr' + \alpha_{rt}\, cdt' \\ cdt = \alpha_{tr}\, dr' + \alpha_{tt}\, cdt', \end{cases} \tag{2.38}$$

where α_{mn} depend only on r. By substitution in shorthand expression (2.36) we get:

$$ds^2 = \gamma^2\, dr^2 - \gamma^{-2}\, c^2 dt^2 =$$

$$= \gamma^2(\alpha_{rr}\, dr' + \alpha_{rt}\, cdt')^2 - \gamma^{-2}(\alpha_{tr}\, dr' + \alpha_{tt}\, cdt')^2$$

$$= (\gamma^2\alpha_{rr}^2 - \gamma^{-2}\alpha_{tr}^2)\, dr'^2 + 2(\gamma^2\alpha_{rr}\alpha_{rt} - \gamma^{-2}\alpha_{tr}\alpha_{tt})\, dr'\, cdt' + (\gamma^2\alpha_{rt}^2 - \gamma^{-2}\alpha_{tt}^2)\, c^2 dt'^2.$$

Equating this interval with $dr'^2 - c^2 dt'^2$ we get the system of equations:

$$\begin{cases} \gamma^2\alpha_{rr}^2 - \gamma^{-2}\alpha_{tr}^2 = 1, \\ \gamma^2\alpha_{rr}\alpha_{rt} - \gamma^{-2}\alpha_{tr}\alpha_{tt} = 0, \\ \gamma^2\alpha_{rt}^2 - \gamma^{-2}\alpha_{tt}^2 = -1. \end{cases}$$

[12] Isaac Newton (1642-1727), English mathematician.

These are three equations with four unknown alphas which means we have an arbitrary parameter; let it be $\alpha_{rr} = \chi \in \mathbb{C}$. It follows from the first equation $\alpha_{tr} = \pm i\gamma\sqrt{1-\gamma^2\chi^2}$. From the third we have $\alpha_{rt} = \pm i\gamma^{-1}\sqrt{1-\gamma^{-2}\alpha_{tt}^2}$, which in the middle gives, in order:

$$\gamma^2\chi(\pm i\gamma^{-1}\sqrt{1-\gamma^{-2}\alpha_{tt}^2}) - \gamma^{-2}(\pm i\gamma\sqrt{1-\gamma^2\chi^2})\alpha_{tt} = 0,$$

$$\gamma\chi\sqrt{1-\gamma^{-2}\alpha_{tt}^2} = \gamma^{-1}\alpha_{tt}\sqrt{1-\gamma^2\chi^2},$$

$$\chi^2(\gamma^2 - \alpha_{tt}^2) = \alpha_{tt}^2(\gamma^{-2} - \chi^2),$$

$$\chi^2\gamma^2 = \alpha_{tt}^2\gamma^{-2},$$

whence it follows $\alpha_{tt} = \pm\gamma^2\chi$ и $\alpha_{rt} = \pm i\gamma^{-1}\sqrt{1-\gamma^2\chi^2}$. When we take only the upper signs, we get the required system. □

This is a much more important result here than at the time when I first published it, or then in the book [8], because it now confirms the two-dimensionality of perception information, otherwise at first glance very controversial.

If we take a closer look at the transformations (2.37) we see that by multiplying the first by γ and the second by $i\gamma^{-1}$ we get:

$$\begin{cases} \gamma\, dr = \gamma\chi\, dr' + \sqrt{1-\gamma^2\chi^2}\, icdt' \\ \gamma^{-1}\, icdt = -\sqrt{1-\gamma^2\chi^2}\, dr' + \gamma\chi\, icdt'. \end{cases} \tag{2.39}$$

By substitutions $dy_1 = \gamma dr$, $dy_4 = \gamma^{-1}icdt$ and $dx_1 = dr'$, $dx_4 = icdt$, then:

$$\cos\varphi = \gamma\chi, \quad \sin\varphi = \sqrt{1-\gamma^2\chi^2}. \tag{2.40}$$

they become ordinary *rotations* because

$$\begin{cases} dy_1 = \cos\varphi\, dx_1 + \sin\varphi\, dx_4, \\ dy_4 = -\sin\varphi\, dx_1 + \cos\varphi\, dx_4. \end{cases} \tag{2.41}$$

The substitutions $dy_0 = \gamma^{-1}cdt$, $dx_0 = cdt$ and $\varphi = i\phi$ gives

$$\begin{cases} dy_1 = \cosh\phi\, dx_1 + \sinh\phi\, dx_0, \\ dy_0 = \sinh\phi\, dx_1 + \cosh\phi\, dx_0, \end{cases} \tag{2.42}$$

and these are hyperbolic rotations, that is, Lorentz transformations. Certainly

$$ds^2 = (\gamma dr)^2 - (\gamma^{-1}cdt)^2 = (dr')^2 - (cdt')^2, \tag{2.43}$$

where $\gamma = 1/\sqrt{1-2GM/rc^2}$ is taken from (2.36).

For the hyperbolic transformations of (2.40) to be "real" (Lorentz equivalent), it suffices $|\gamma\chi| \leq 1$. Then from (2.42) and $\beta = i\tanh\phi$ we get

$$\gamma\chi = \frac{1}{\sqrt{1-\beta^2}}, \quad \beta = \frac{v}{c}, \tag{2.44}$$

which reveals the nature of the second coefficient (χ). It is a dimensionless number that determines the initial velocity of a free-fall body in a gravitational field. In other words, the number χ defines the height from which a given body began to fall.

For example, in the case of zero initial velocity at infinity ($v \to 0$ when $r \to \infty$), from $ma = F_g$ we calculate $v^2 = 2GM/r$, and hence $\gamma\chi = 1$. The transformations (2.37) become $\gamma\, dr = dr'$ and $\gamma^{-1} cdt = cdt'$ meaning that proper values $dr_0 = dr'$ and $dt_0 = dt'$, the points in still, give:

$$dr = dr_0\sqrt{1 - \frac{2GM}{rc^2}}, \quad dt = \frac{dt_0}{\sqrt{1 - \frac{2GM}{rc^2}}} \tag{2.45}$$

relative to the falling body. And that result is known.

When we see a glass on a table at one moment, it means that its position is most likely at that moment. It will appear in the same place also in the next moment and the next, until any force do not acts on it. We can move the glass from the table by hand, which means that the hand, therefore the force, changes the probabilities. Changing probabilities means changing the state of uncertainty, equivalent to changing the uncertainty by generating information from it. Because of the law of keeping information and generating information out of uncertainty, we must consider uncertainty as a type of information, and in addition, the emission of information by the act of changing uncertainty. Therefore, the two, force and information, should be regarded as related physical sizes.

2.4 Euler–Lagrange equation

Perception information (2.2) is the sum of "freedoms" (quantities of options), and each sum is the product of a corresponding "ability" of the subject and objective "difficulties". Since "freedom" is some information, its exponent is some amount of options, and the reciprocal of that exponent is some mean value of the probabilities of the outcomes of those options. We know that more likely events occur more often, and then we know that natural processes spontaneously flow toward less emission of information, which means that nature tends to evolve into states with as little "freedom" as possible. This is the *principle of minimalism.*

As the perception information is formally the sum of the products of the corresponding pairs given by two n-tuples, two series with $n \in \mathbb{N}$ items each, it is then and the scalar product of the vector. The components (items) of these two vectors are observables (the first measures the intelligence of the individual, the second hierarchy around), and they can always be reduced to one plane with only two "observables". I say two alleged observables, because we leave aside the physical measurability of the values of reduced components.

Candidacy for such perception information will be given by *Lagrangian*[13], the difference between kinetic and potential energy, otherwise very familiar to theoretical physics. In Newtonian mechanics, we define Lagrangian with

$$\mathcal{L} = E_k - E_p, \tag{2.46}$$

where E_k is the total kinetic energy of the observed system (body) and E_p is its total potential energy. As we know, the product of energy and time is the physical *action* that we will refer to here as $\mathcal{S}$.

Example 2.4.1 (Spring)**.** *Define Lagrangian of elastic spring.*

[13] see [16]

Solution. Take the simple case of mass m at the end of a spring along x-axis. The kinetic and potential energy of the spring are:

$$E_k = \frac{m\dot{x}^2}{2}, \quad E_p = \frac{kx^2}{2},$$

so the spring's Lagrangian is:

$$\mathcal{L} = E_k - E_p = \frac{m\dot{x}^2}{2} - \frac{kx^2}{2}. \tag{2.47}$$

The derivation of length by time is speed $v = \dot{x}$, □

For partial derivations of the Lagrangian (2.47), in terms of speed and path we find:

$$\frac{\partial \mathcal{L}}{\partial \dot{x}} = m\dot{x}, \quad \frac{\partial \mathcal{L}}{\partial x} = -kx.$$

According to Newton's law force is proportional to mass and acceleration, here

$$m\ddot{x} = -kx,$$

and hence even more general equations of motion

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{x}}\right) = \frac{\partial \mathcal{L}}{\partial x}. \tag{2.48}$$

This is the known *Euler-Lagrange equation.* It express the condition that the Lagrangian $\mathcal{L}$ be such that its integral over time, the action $\mathcal{S}$, becomes maximal or minimal.

There are many known ways to derivate Equation (2.48), but at least one should be specified to complete the text. Some are more mathematically correct, others are more general, thirds are more popular, and the following[14] may be the most appropriate for this book.

Theorem 2.4.2 (Euler-Lagrange equation)**.** *Prove that equation*

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} = \frac{\partial \mathcal{L}}{\partial q}, \tag{2.49}$$

where $q = q(t)$ is a coordinate variable with time t, $\dot{q}$ its derivation by time, the function $\mathcal{L} = \mathcal{L}(q, \dot{q}, t)$ is Lagrangian and its integral over time, the action $\mathcal{S}$, is maximal or minimal.

Dokaz. We change the coordinate $q(t)$ by a small variation of $\eta(t)$, to infinitesimal, with boundary conditions $\eta(t_1) = \eta(t_2) = 0$. The condition for extremes is:

$$\frac{d}{d\epsilon}\mathcal{S} = \frac{d}{d\epsilon}\int_{t_1}^{t_2} \mathcal{L}\left(q(t) + \epsilon\eta(t), \dot{q}(t) + \epsilon\dot{\eta}(t), t\right) dt = 0,$$

$$\int_{t_1}^{t_2} \left(\eta\frac{\partial \mathcal{L}}{\partial q} + \dot{\eta}\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) dt = 0,$$

$$\int_{t_1}^{t_2} \eta\frac{\partial \mathcal{L}}{\partial q}\, dt + \left(\int_{t_1}^{t_2} \frac{\partial \mathcal{L}}{\partial \dot{q}}\, d\eta\right) = 0,$$

[14] Euler-Lagrange-Equation: `https://martin-ueding.de/`

$$\int_{t_1}^{t_2} \eta \frac{\partial \mathcal{L}}{\partial q}\, dt + \left(\frac{\partial \mathcal{L}}{\partial \dot{q}} \eta(t) \Big|_{t_1}^{t_2} - \int_{t_1}^{t_2} \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} \eta\, dt \right) = 0,$$

$$\int_{t_1}^{t_2} \eta \frac{\partial \mathcal{L}}{\partial q}\, dt - \int_{t_1}^{t_2} \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} \eta\, dt = 0,$$

$$\int_{t_1}^{t_2} \left(\frac{\partial \mathcal{L}}{\partial q} - \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}} \right) \eta(t)\, dt = 0.$$

This is true for every $\eta(t)$, so the expression in parentheses is zero, which is equation (2.49). □

Consistently with the previous considerations, we continue with the principle of minimalism in the movement of celestial bodies in gravity. Schwarzschild metric (2.36), or some other of Einstein's equations, can be summarized as a general expression

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu, \tag{2.50}$$

where $g_{\mu\nu}$ $(\mu, \nu = 1, 2, 3, 4)$ is a twice covariant metric tensor, dx^μ are the differentials of the contravariant coordinates, whereby according to Einstein's the tensor convention summation compiles to the repeated lower and upper indexes. The interval ds is the shortest path between the given points in the 4-dim space-time theory of relativity.

The integral of that shortest path is the action

$$\mathcal{S} = \lambda \int_a^b ds, \tag{2.51}$$

where λ is a currently unknown constant. The Lagrangian is at a given point

$$\mathcal{L} = \lambda \dot{\mathcal{S}}, \tag{2.52}$$

where the point above the interval means derivation by time, often by the system's proper (own) time of the given place.

Example 2.4.3 (Lorentz Lagrangian)**.** *Find the Lagrangian for metrics (2.33).*

Solution. Lorentz interval

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2 dt^2,$$

is valid in the absence of gravity and in Descartes' rectangular coordinates. By (2.50) then $x^1 = x$, $x^2 = y$, $x^3 = z$ and $x^4 = ict$, while $g_{11} = g_{22} = g_{33} = 1$, $g_{44} = -1$, and all other coefficients of the metric tensor are zero. The spatial length is

$$dl^2 = dx^2 + dy^2 + dz^2,$$

so the 4-interval is reduced to

$$ds^2 = dl^2 - c^2 dt^2 = -\left(1 - \frac{v^2}{c^2}\right) c^2 dt^2,$$

where $v = dl/dt$ is the velocity of motion of a given body (material points). For the low speed v, due to $c \to \infty$, the Lagrangian must go to the classical $\mathcal{L} = mv^2/2$, so that

$$\mathcal{L} = -mc^2 \sqrt{1 - \frac{v^2}{c^2}} \approx -mc^2 + \frac{mv^2}{2} = E_k - E_p,$$

which means that $\lambda = imc$. □

In the general case of the gravitational field, it is Lagrangian

$$\mathcal{L} = imc\sqrt{g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}. \tag{2.53}$$

It is a real negative number because the expression under the root is always negative ($i^2 = -1$). The latter means that the potential energy of a body (particle) in the gravitational field is always greater than its kinetic energy. If we worked the previous example in the general case, in the weaker fields we call Newton's, we would compare (2.53) with the well-known nonrelativistic Lagrangian ($\mathcal{L} = E_k - E_p$), so with

$$\mathcal{L} = \frac{mv^2}{2} - m\phi, \tag{2.54}$$

where ϕ is a certain function of coordinates and time that characterizes the field and is called *gravitational potential*, m is the mass of the particle, v is its velocity. The corresponding equations of motion are

$$\dot{\mathbf{v}} = -\text{grad } \phi. \tag{2.55}$$

No mass or other property of the particle is required to define its acceleration in the gravitational field.

We will now derive the general differential equations of motion, following the principle of least action. We will obtain equations known in classical physics, but pay attention to the unusual way of this derivation because it is based on the relativistic Lagrangian (2.53). The *Christoffel*[15] symbols for writing down these equations, which we find by the way, are, of course, exactly the same as those otherwise known to us from *Riemann's*[16] geometry or Einstein's gravity.

The metric coefficients $g_{\mu\nu}$ depend on the location of the given particle, but not on its velocity. In other words, they can be changed with x^α, but not with dx^α/dt. Thus, differentiating $\mathcal{L}$ by $\dot{x}^\sigma = dx^\sigma/dt$ gives:

$$\frac{\partial\mathcal{L}}{\partial\dot{x}^\sigma} = wg_{\mu\nu}\frac{\partial}{\partial\dot{x}^\sigma}\left(\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}\right) =$$

$$= wg_{\mu\nu}\left[\left(\frac{\partial}{\partial\dot{x}^\sigma}\frac{dx^\mu}{dt}\right)\frac{dx^\nu}{dt} + \frac{dx^\mu}{dt}\left(\frac{\partial}{\partial\dot{x}^\sigma}\frac{dx^\nu}{dt}\right)\right]$$

$$= wg_{\mu\nu}\left(\delta^\mu_\sigma\frac{dx^\nu}{dt} + \frac{dx^\mu}{dt}\delta^\nu_\sigma\right), \quad \dot{x}^\sigma = \frac{dx^\sigma}{dt}$$

$$= w\left(g_{\nu\sigma}\frac{dx^\nu}{dt} + g_{\mu\sigma}\frac{dx^\mu}{dt}\right)$$

$$= w\cdot 2g_{\mu\sigma}\frac{dx^\mu}{dt}.$$

where

$$w = \frac{imc}{2\sqrt{g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}} = \frac{imc}{2\frac{ds}{dt}} = \frac{imc}{2}\frac{dt}{ds}.$$

[15]Elwin Christoffel (1829-1900), German mathematician.
[16]Bernhard Riemann (1826-1866), German mathematician.

Hence:

$$\frac{\partial \mathcal{L}}{\partial \dot{x}^\sigma} = \frac{imc\, g_{\mu\sigma}}{\sqrt{g_{\mu\nu}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt}}}\frac{dx^\mu}{dt} = imc\, g_{\mu\sigma}\frac{dx^\mu}{ds}. \tag{2.56}$$

The usual derivative with respect to t is:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} = imc\frac{d}{dt}\left(g_{\mu\sigma}\frac{dx^\mu}{ds}\right)$$

$$= imc\left(\frac{dg_{\mu\sigma}}{dt}\frac{dx^\mu}{ds} + g_{\mu\sigma}\frac{d^2x^\mu}{dsdt}\right),$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}} = imc\left(\frac{\partial g_{\mu\sigma}}{\partial x^\alpha}\frac{dx^\alpha}{dt}\frac{dx^\mu}{ds} + g_{\mu\sigma}\frac{d^2x^\mu}{dsdt}\right). \tag{2.57}$$

Because only $g_{\mu\nu}$ explicitly depends on x^σ, we have the following:

$$\frac{\partial \mathcal{L}}{\partial x^\sigma} = w\frac{\partial g_{\mu\nu}}{\partial x^\sigma}\frac{dx^\mu}{dt}\frac{dx^\nu}{dt} = \frac{imc}{2}\frac{\partial g_{\mu\nu}}{\partial x^\sigma}\frac{dx^\mu}{dt}\frac{dx^\nu}{ds}. \tag{2.58}$$

Well, we can complete the Euler-Lagrangian equations and transform them:

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{x}^\sigma} - \frac{\partial \mathcal{L}}{\partial x^\sigma} = 0,$$

$$imc\left(\frac{\partial g_{\mu\sigma}}{\partial x^\alpha}\frac{dx^\alpha}{dt}\frac{dx^\mu}{ds} + g_{\mu\sigma}\frac{d^2x^\mu}{dsdt}\right) - \frac{imc}{2}\frac{\partial g_{\mu\nu}}{\partial x^\sigma}\frac{dx^\mu}{dt}\frac{dx^\nu}{ds} = 0,$$

$$g_{\mu\sigma}\frac{d^2x^\mu}{dsdt} + \frac{\partial g_{\mu\sigma}}{\partial x^\alpha}\frac{dx^\alpha}{dt}\frac{dx^\mu}{ds} - \frac{1}{2}\frac{\partial g_{\mu\nu}}{\partial x^\sigma}\frac{dx^\mu}{dt}\frac{dx^\nu}{ds} = 0,$$

$$g_{\mu\sigma}d^2x^\mu + \frac{\partial g_{\mu\sigma}}{\partial x^\alpha}dx^\alpha dx^\mu - \frac{1}{2}\frac{\partial g_{\mu\nu}}{\partial x^\sigma}dx^\mu dx^\nu = 0. \tag{2.59}$$

These are general equations of motion in a space of arbitrary metric.

The same equations can also be expressed using *Christoffel symbols*. Christoffel symbols are defined with respect to the base vectors of a given coordinate system:

$$\frac{\partial \mathbf{e}_\mu}{\partial x^\nu} = \Gamma^\sigma_{\mu\nu}\mathbf{e}_\sigma. \tag{2.60}$$

It is not obvious, but looking at the second covariant scalar derivation, one can prove:

$$\Gamma^\sigma_{\mu\nu} = \Gamma^\sigma_{\nu\mu}. \tag{2.61}$$

Christoffel symbols are symmetrical on the replace of the lower two indexes.

As we know, the base vectors $\mathbf{e}_\sigma$ are defined such that

$$ds^2 = d\mathbf{s}\cdot d\mathbf{s} = (dx^\mu\mathbf{e}_\mu)\cdot(dx^\nu\mathbf{e}_\nu) =$$

$$= \mathbf{e}_\mu\cdot\mathbf{e}_\nu\, dx^\mu dx^\nu = g_{\mu\nu}dx^\mu dx^\nu.$$

In a local plane frame with Descartes' rectangular coordinates, all the base vectors $\mathbf{e}_\sigma$ are constants, so all Christoffel symbols must be zero.

That is why in an arbitrary metric we have:

$$\frac{\partial g_{\mu\nu}}{\partial x^\alpha} = \frac{\partial(\mathbf{e}_\mu \cdot \mathbf{e}_\nu)}{\partial x^\alpha} = \frac{\partial \mathbf{e}_\mu}{\partial x^\alpha} \cdot \mathbf{e}_\nu + \mathbf{e}_\mu \cdot \frac{\partial \mathbf{e}_\nu}{\partial x^\alpha}$$

$$= \Gamma^\sigma_{\alpha\mu} \mathbf{e}_\sigma \cdot \mathbf{e}_\nu + \mathbf{e}_\mu \cdot \Gamma^\sigma_{\alpha\nu} \mathbf{e}_\sigma.$$

The partial derivations of the metric tensor are not tensors, so Christoffel symbols are not tensors.

The general equations of motion now become:

$$g_{\mu\sigma} d^2 x^\mu + \Gamma^\beta_{\alpha\mu} g_{\beta\sigma} dx^\alpha dx^\mu + \Gamma^\beta_{\alpha\mu} g_{\mu\beta} dx^\alpha dx^\mu - \frac{1}{2}\Gamma^\beta_{\sigma\mu} g_{\beta\nu} dx^\mu dx^\nu - \frac{1}{2}\Gamma^\beta_{\sigma\nu} g_{\mu\beta} dx^\mu dx^\nu = 0.$$

In the second and third items, we change the addition of α with ν. Then we get:

$$2 g_{\mu\sigma} d^2 x^\mu + (2\Gamma^\beta_{\nu\mu} g_{\beta\sigma} + 2\Gamma^\beta_{\nu\sigma} g_{\mu\beta} - \Gamma^\beta_{\sigma\mu} g_{\beta\nu} - \Gamma^\beta_{\sigma\nu} g_{\mu\beta}) dx^\mu dx^\nu = 0.$$

Due to the aforementioned symmetry, after shortening we obtain:

$$g_{\mu\sigma} d^2 x^\mu + \Gamma^\beta_{\mu\nu} g_{\beta\sigma} dx^\mu dx^\nu = 0. \tag{2.62}$$

These equations of motion can be simplified by multiplying by a contravariant metric tensor:

$$g^{\sigma\alpha} g_{\mu\sigma} d^2 x^\mu + g^{\sigma\alpha} \Gamma^\beta_{\mu\nu} g_{\beta\sigma} dx^\mu dx^\nu = 0,$$

$$\delta^\alpha_\mu d^2 x^\mu + \delta^\alpha_\beta \Gamma^\beta_{\mu\nu} dx^\mu dx^\nu = 0,$$

$$d^2 x^\alpha + \Gamma^\alpha_{\mu\nu} dx^\mu dx^\nu = 0. \tag{2.63}$$

These are the required general equations of motion expressed by Christoffel symbols of the second kind.

2.5 Einstein's general equations

We have seen that the classical Lagrangian builds on the relativistic metric and gives exactly the same geodesic lines that are familiar to us from the Riemannian geometry. These are the curved lines that follow the shortest distances in non-Euclidean geometries, and which in Einstein's gravitational fields are events (points of 4-dim space-time) of the "free fall" of satellites in weightless states. Now these are the paths of least energy consumption and at the same time the least consumption of information.

For the sake of consistency, we will prove below that the same *principle of minimalism* on which the use of Lagrangians is based can also derive Einstein's famous *general equations* in exactly the same form as he discovered them by considering inertial motions and influences of matter, that is, energy on space-time. In all these and similar considerations, the term "principle of least action" can always be replaced by "principle of least information" and vice versa.

As is known, Einstein made his general field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G}{c^4} T_{\mu\nu}, \tag{2.64}$$

formed in 1916 "directly", noting that the left side of equality should represent the geometry of space (determined by the tensors of curvature and metric) and the right side of matter (determined by the energy tensor). Only much later, they were obtained (in exactly the same form) by the principle of least action.

In the book Space-Time, I explained Einstein's method, cited a non-relativistic derivation of the metric-symmetric gravity field[17] and pointed to the great degree of approximation of that with the Schwarzschild solution. Here, I reiterated part of one of my older contributions, [16], in which from the Euler–Lagrange equation (in a nonrelativistic way) are derived Christoffel symbols and general equations of motion, so now we need to complete this discussion by demonstrating the consistency of this method in general theory of relativity. You can find more interesting evidence about it on the site dedicated to Einstein[18].

In the absence of matter and energy (in a vacuum), we usually guess the action as

$$\mathcal{S} = \int \mathcal{L}\, d^4V, \tag{2.65}$$

where now is taken the integral of the Lagrangian $\mathcal{L}$, the scalar of the energy change density, per d^4V element of the infinitesimal 4-volume. In local coordinates of $x^{\mu'}$ 4-dimensional Minkowski space-time, such as the element of volume is

$$d^4V = dx^{1'}dx^{2'}dx^{3'}dx^{4'} = (\det J^{\mu'}_{\nu})dx^1dx^2dx^3dx^4 = (\det J^{\mu'}_{\nu})d^4x,$$

where $\det J^{\mu'}_{\nu}$ is the Jacobian of the given transformation of coordinates. It is shown that, in the general case, the metric tensor is subject to coordinate transformations

$$g_{\mu\nu} = J^{\alpha'}_{\mu} J^{\beta'}_{\nu} g_{\alpha'\beta'}, \tag{2.66}$$

so if g is determinant of a metric tensor matrix of type 4×4, then $g = -(\det J)^2$ and therefore $\det J = \sqrt{-g}$. It's a 4-volume element

$$d^{(4)}V = \sqrt{-g}\, d^4x. \tag{2.67}$$

As we know (Bianchi Identities, [17]), the simplest Lagrangian $\mathcal{L}$ which is a scalar function of the metric $g_{\mu\nu}$ and whose derivation is Ricci's scalar R (which is derived from the Riemannian tensor $R^{\mu}_{\alpha\beta\gamma}$, see [9]) is exactly that scalar, $\mathcal{L} = R$.

Hence the Einstein-Hilbert action

$$S = \int_V \mathcal{L}\sqrt{-g}\, d^4x = \int_V R\sqrt{-g}\, d^4x. \tag{2.68}$$

It can also be obtained in form

$$S = \frac{1}{16\pi G}\int (R - 2\Lambda)\sqrt{-g}\, d^4x, \tag{2.69}$$

which includes units of measure with $G = 6.67408 \times 10^{-11}\mathrm{m}^3\mathrm{kg}^{-1}\mathrm{s}^{-2}$ universal gravitational constant and cosmological constant Λ. Both forms are translated into given.

The following is to obtain Einstein's equation (2.64) from action (2.69). As usual, to vary the action such that $\delta S = 0$, we find:

$$\delta S = \delta \int R\sqrt{-g}\, d^4x = \delta \int g^{\mu\nu}R_{\mu\nu}\sqrt{-g}\, d^4x = \int \delta(g^{\mu\nu}R_{\mu\nu}\sqrt{-g})\, d^4x =$$

[17] [8], (1.126) p. 59 et seq.

[18] Einstein Relatively Easy: `http://einsteinrelativelyeasy.com/`

$$= \int (\delta g^{\mu\nu} R_{\mu\nu}\sqrt{-g} + \delta R_{\mu\nu} g^{\mu\nu}\sqrt{-g} + R_{\mu\nu} g^{\mu\nu}\delta\sqrt{-g})\, d^4x$$

$$= \int (\delta g^{\mu\nu} R_{\mu\nu}\sqrt{-g} + \delta R_{\mu\nu} g^{\mu\nu}\sqrt{-g} + R\delta\sqrt{-g})\, d^4x.$$

I quote this more for validation of method and training than for doubt of result. Knowing that it is

$$\delta\sqrt{-g} = -\frac{1}{2}\sqrt{-g} g_{\mu\nu}\delta g^{\mu\nu},$$

we find:

$$\delta S = \int (\delta g^{\mu\nu} R_{\mu\nu}\sqrt{-g} - \frac{1}{2}\sqrt{-g} g_{\mu\nu}\delta g^{\mu\nu} R + \delta R_{\mu\nu} g^{\mu\nu}\sqrt{-g})\, d^4x$$

$$= \int (R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R)\delta g^{\mu\nu}\sqrt{-g}\, d^4x + \int g^{\mu\nu}\delta R_{\mu\nu}\sqrt{-g}\, d^4x.$$

Putting $\delta S = 0$ and knowing that $\delta g^{\mu\nu}$ is completely arbitrary, we obtain the Einstein equation in vacuum

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0, \tag{2.70}$$

if and only if we can determine that the other member is waste, i.e. that

$$\int g^{\mu\nu}\delta R_{\mu\nu}\sqrt{-g}\, d^4x = 0. \tag{2.71}$$

For that part of the proof, we start from the Riemannian curve tensor $R^{\alpha}_{\mu\sigma\nu}$, which corresponds to the difference of parallel transmission of vectors in two opposite directions of a closed path. Another interpretation of the Riemannian tensor is the relative acceleration of adjacent particles in free fall.

We will not derivate this tensor

$$R^{\alpha}_{\mu\sigma\nu} = \partial_\sigma \Gamma^{\alpha}_{\mu\nu} - \partial_\nu \Gamma^{\alpha}_{\mu\sigma} + \Gamma^{\alpha}_{\sigma\gamma}\Gamma^{\gamma}_{\mu\nu} - \Gamma^{\alpha}_{\nu\gamma}\Gamma^{\gamma}_{\nu\sigma} \tag{2.72}$$

or to check the last two otherwise known claims, but I'll immediately proceed to the continuation of our evidence. By contracting this tensor by the first and third indexes (putting $\sigma = \alpha$ and adding by this index), we get the Ricci tensor

$$R_{\mu\nu} = R^{\alpha}_{\mu\alpha\nu} = \partial_\alpha \Gamma^{\alpha}_{\mu\nu} - \partial_\nu \Gamma^{\alpha}_{\mu\alpha} + \Gamma^{\alpha}_{\alpha\gamma}\Gamma^{\gamma}_{\mu\nu} - \Gamma^{\alpha}_{\nu\gamma}\Gamma^{\gamma}_{\mu\alpha}. \tag{2.73}$$

By varying we get

$$\delta R_{\mu\nu} = \partial_\alpha \delta\Gamma^{\alpha}_{\mu\nu} - \partial_\nu \delta\Gamma^{\alpha}_{\mu\alpha} + \delta\Gamma^{\alpha}_{\alpha\gamma}\Gamma^{\gamma}_{\mu\nu} + \Gamma^{\alpha}_{\alpha\gamma}\delta\Gamma^{\gamma}_{\mu\nu} - \delta\Gamma^{\alpha}_{\nu\gamma}\Gamma^{\gamma}_{\mu\alpha} - \Gamma^{\alpha}_{\nu\gamma}\delta\Gamma^{\gamma}_{\mu\alpha}. \tag{2.74}$$

The first two summands suggest that there may be differences between the two covariance derivatives.

Higher-order covariant derivatives of tensor, as we know, are obtained by taking the partial derivation of the tensor, then adding the Christoffel symbol $\Gamma^{\alpha}_{\gamma\beta}$ to each of the above indices and subtracting the symbol $\Gamma^{\gamma}_{\alpha\beta}$ to each index below. So, for example, we have the covariance derivation

$$\Delta_\beta T^{\mu}_{\nu} = \frac{\partial T^{\mu}_{\nu}}{\partial x^\beta} + T^{\alpha}_{\nu}\Gamma^{\mu}_{\alpha\beta} - T^{\mu}_{\alpha}\Gamma^{\alpha}_{\nu\beta}, \tag{2.75}$$

of second order tensors T^{μ}_{ν}. When both indices of this tensor (twice covariant) are lower, than the (first) addition goes to subtraction, and if both indices are upper (contravariant) we

have two additions. In doing so, the indices are adjusted in such a way that the upper and lower indices (in the products) are equal in order to apply the tensor summing convention.

Using the covariant derivation, we get the expression

$$\Delta_\alpha(\delta\Gamma^\alpha_{\mu\nu}) - \Delta_\nu(\delta\Gamma^\alpha_{\mu\alpha}) = \delta R_{\mu\nu}, \tag{2.76}$$

referred to as Palatini equality. We turn this one to the left side of equality (2.71) and, after arranging, we find

$$\int g^{\mu\nu}\delta R_{\mu\nu}\sqrt{-g}\,d^4x = \int \Delta_\alpha(g^{\mu\nu}\delta\Gamma^\alpha_{\mu\nu} - g^{\mu\alpha}\delta\Gamma^\nu_{\mu\nu})\sqrt{-g}\,d^4x. \tag{2.77}$$

In the expression in parentheses, the indices μ and ν are canceled, so that it is a first-order tensor

$$A^\alpha = g^{\mu\nu}\delta\Gamma^\alpha_{\mu\nu} - g^{\mu\alpha}\delta\Gamma^\nu_{\mu\nu}, \tag{2.78}$$

so we are left

$$\int g^{\mu\nu}\delta R_{\mu\nu}\sqrt{-g}\,d^4x = \int_V \Delta_\alpha A^\alpha\sqrt{-g}\,d^4x, \tag{2.79}$$

which can be reduced to the surface integral of the divergence theorem[19], which disappears because variations are assumed to disappear on the surface V. So we finally get Einstein's equation (2.70) for the vacuum.

To prove the general equation (2.64), for space that is not empty but contains matter, the action of S_{AH} should be added to Einstein-Hilbert action S_M, so the total action

$$S = kS_{AH} + S_M. \tag{2.80}$$

Now it is:

$$\delta S_{AH} = \int \left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)\sqrt{-g}\delta g^{\mu\nu}d^4x = \int \frac{\delta S_{AH}}{\delta g^{\mu\nu}}\delta g^{\mu\nu}\,d^4x,$$

$$\frac{\delta S_{AH}}{\delta g^{\mu\nu}} = \left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right)\sqrt{-g},$$

$$\frac{1}{\sqrt{-g}}\frac{\delta S_{AH}}{\delta g^{\mu\nu}} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R,$$

so varying the total action gives:

$$\frac{1}{\sqrt{-g}}\frac{\delta S}{\delta g^{\mu\nu}} = \frac{1}{\sqrt{-g}}\frac{k\delta S_{AH}}{\delta g^{\mu\nu}} + \frac{1}{\sqrt{-g}}\frac{\delta S_M}{\delta g^{\mu\nu}} = k\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) + \frac{1}{\sqrt{-g}}\frac{\delta S_M}{\delta g^{\mu\nu}} = 0.$$

By arranging, we get:

$$k\left(R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R\right) = -\frac{1}{\sqrt{-g}}\frac{\delta S_M}{\delta g^{\mu\nu}},$$

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\frac{1}{k\sqrt{-g}}\frac{\delta S_M}{\delta g^{\mu\nu}}.$$

Defining the energy-momentum tensor with

$$T_{\mu\nu} = \frac{2}{\sqrt{-g}}\frac{\delta S_M}{\delta g^{\mu\nu}} \tag{2.81}$$

[19] Divergence theorem: `https://en.wikipedia.org/wiki/Divergence_theorem`

and putting $k = c^4/2(8\pi G)$ we obtain the required Einstein equations (2.64).

Einstein did not derive his equations from the principle of least action, but from the same results we can now see that the gravitational field is a space of actions, and then it is also a space of exchange of physical information, that is, a space of communication. The geodesic lines, for which we have now proved that the principle of the least action is valid, for Einstein were trajectories of free fall. Combining the two into a theory of (physical) information, we can now determine that the emission of information on proper (its own) trajectory is the smallest possible, and that the inherent probability of its path, as seen by free-falling bodies, is the greatest.

We note that this agrees with the previous observation (Maxwell's demon) that the thermodynamic circular process when it loses energy becomes irreversible, loses information and entropy grows. Relative to the free-fall body, the entropy of air in a room on earth is smaller because the gas molecules closer to the floor are distributed more densely. Some (unequal) relative entropy reduction exists for the "rooms" on either side of the path, lower and upper. Because the entropy of this air is relatively smaller, the emission of information to the outside is relatively higher, and because of the principle of minimalism of information, the body remains in its free fall and does not spontaneously switch to such other states.

Another example of consent is the Compton Effect. Because by collision and change of the path, the wavelength of the photon increases, its smear on that new path increases, the probability density decreases, so we say that the photon did not want to spontaneously divert to the path of less probability (from its own point of view), or turn away in states of higher emissions information. Consequently, because nature is skimping with information, therefore we have the law of inertia.

2.6 Schrödinger equation

Let's go back to perception information (2.3) again and look at one of its item L_k which we call "freedom". Each of these summand is information about a wave phenomenon and, on the other hand, logs of some probabilities. We then wrote this probability in the form $P_k = \exp(L_k)$, choosing for the base the Euler number ($e \approx 2.718$), thus we just decided on the unit of information. In order to include wave phenomena, we will have to present information using a complex number, and here is why.

Basically we express periodic phenomena with the sinusoids[20]

$$y = A \cdot \sin(\omega t - \delta). \tag{2.82}$$

From these we obtain cosineusoids, say from:

$$\cos\theta = \sin(\frac{\pi}{2} - \theta), \quad \cos\theta = \pm\sqrt{1 - \sin^2\theta}, \tag{2.83}$$

and similar formulas, and we further obtain other trigonometric and wave functions. Fourier's development of the[21] function in the order of sines and cosines generalizes this idea.

It is assumed that we know some mathematical basics like

$$\begin{cases} e^{i\theta} = \cos\theta + i\sin\theta, \\ e^{i\theta_1}e^{i\theta_2} = \cos(\theta_1 + \theta_2) + i\sin(\theta_1 + \theta_2), \end{cases} \tag{2.84}$$

[20] I will use [12] for greater persuasion.

[21] Fourier series: `https://en.wikipedia.org/wiki/Fourier_series`

where $i^2 = -1$, as well as the hypothesis of Louis de Broglie[22] of 1924, that any moving particle can be joined by a wave, which has been largely confirmed and accepted in physics.

We transcribe the formal sine function (2.82) into the form

$$y = A \sin \frac{2\pi\nu}{\lambda}\left(t - \frac{x}{v}\right), \tag{2.85}$$

which is more physical. The wave function associated with a particle moving along x-axis is given by

$$\psi = Ae^{-i\omega\left(t-\frac{x}{v}\right)}, \tag{2.86}$$

where A is the amplitude of the oscillations, ν is the frequency of the wave, $\omega = 2\pi\nu$ is the angular frequency, λ is the wavelength, x and t are the place and moment, and $v = \nu\lambda$ is the velocity of the wave.

The total energy of oscillation is

$$E = h\nu = 2\pi\hbar\nu, \tag{2.87}$$

where h is the Planck constant and $\hbar = h/2\pi$, so $\nu = E/2\pi\hbar$. According to Louis de Broglie's hypothesis $\lambda = h/p = 2\pi\hbar/p$ where p is the momentum of a particle, so (2.86) becomes

$$\psi = Ae^{-\frac{i}{\hbar}(Et-xp)}. \tag{2.88}$$

This is a mathematical representation of the free particle of the total energy E of momentum p moving in the direction x-axis.

Total energy is the sum of kinetic and potential ($E = E_k + E_p$), that is:

$$E = \frac{1}{2}mv^2 + E_p = \frac{m^2v^2}{2m} + E_p = \frac{p^2}{2m} + E_p, \tag{2.89}$$

so

$$E\psi = \frac{p^2}{2m}\psi + E_p\psi. \tag{2.90}$$

By partial differentiation (2.88) by x, we obtain

$$\frac{\partial\psi}{\partial x} = Ae^{-\frac{i}{\hbar}(Et-xp)}\frac{ip}{\hbar}, \tag{2.91}$$

and by differentiating again by x

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{p^2}{\hbar^2}Ae^{-\frac{i}{\hbar}(Et-xp)}. \tag{2.92}$$

Using (2.88) we write this result

$$\frac{\partial^2\psi}{\partial x^2} = -\frac{p^2\psi}{\hbar^2}, \tag{2.93}$$

that is

$$p^2\psi = -\hbar^2\frac{\partial^2\psi}{\partial x^2}. \tag{2.94}$$

[22] Louis de Broglie (1892-1987), French physicist.

Differentiating (2.88) by t, we get

$$\frac{\partial \psi}{\partial t} = Ae^{-\frac{i}{\hbar}(Et-xp)}\frac{-iE}{\hbar}. \tag{2.95}$$

Reuse (2.88) gives

$$\frac{\partial \psi}{\partial t} = \frac{-iE}{\hbar}\psi, \tag{2.96}$$

respectively

$$E\psi = -\frac{\hbar}{i}\frac{\partial \psi}{\partial t}. \tag{2.97}$$

Equation (2.90) becomes

$$-\frac{\hbar}{i}\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + E_p\psi. \tag{2.98}$$

Substituting (2.94) and (2.97) to (2.90) yields

$$\frac{\hbar}{i}\frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} - E_p\psi. \tag{2.99}$$

We obtain the one-dimensional time-dependent *Schrödinger equation.*

The same can be written in three dimensions as

$$\frac{\hbar}{i}\frac{\partial \psi}{\partial t} = \frac{\hbar^2}{2m}\nabla^2\psi - E_p\psi, \tag{2.100}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \tag{2.101}$$

is *Laplace*[23] operator.

This is about the toughest that can be found from all the proofs offered to date of the famous Schrödinger equation, first exhibited in 1926, at first much more extensive and much more complicated. The advantage of this contribution is also in the clearer connection of the wave function ψ with the perception information, I hope.

2.6.1 Generally about solutions

We obtain the time-dependent Schrödinger equation (2.100) which is often written in the following form

$$i\hbar\frac{\partial}{\partial t}\Psi(\mathbf{r},t) = \left[\frac{-\hbar^2}{2m}\nabla^2 + V(\mathbf{r},t)\right]\Psi(\mathbf{r},t), \tag{2.102}$$

where $\Psi(\mathbf{r},t)$ is a wave function determined by the position vector $\mathbf{r}$ at the time t. Note that when $\Psi_1, \Psi_2, \ldots$ are solutions of this differential equation, then also

$$\Psi = \alpha_1\Psi_1 + \alpha_2\Psi_2 + \ldots \tag{2.103}$$

are solution, where $\alpha_1, \alpha_2, \cdots \in \mathbb{C}$ are arbitrary constants, and the items are a natural number or infinite. It is easy to further demonstrate[24] that such solutions constitute a vector space, especially the Hilbert vector space.

[23] Pierre-Simon Laplace (1749-1827), French mathematician.

[24] You have the definition of vector space in "Quantum Mechanics" [4].

As we know, matrices (of the same type) also make up vector spaces. Matrix-columns (or matrix-rows) are equivalent to series, and these can be interpreted by classical radius-vectors of point positions, including oriented lengths. Linear array operators are matrices with as many columns (rows) as the series has components. They act on their vectors multiplying them from the right (left) to form some new vectors, copies of the original.

Because of the properties of the derivative, $(cf)' = cf'$, when the matrix is constant, when all its coefficients are constants, and it acts on a vector that is a solution of the Schrödinger equation, then the result is a solution of that equation. In general, a constant linear operator will map each solution of the Schrödinger equation to some solution of the same equation, simply because that equation is linear.

The meanings of these simple algebraic properties for quantum mechanics are invaluable. The observable, measurable quantities we want to observe represent the coordinate axes, the quantum states which, due to the discreteness of the information, are in the end always some particles represented by vectors, and quantum evolutions by operators. The vector is standardized (unit) because it represents superposition, it defines the probability distributions of the occurrence of the quantum state by given observables, and the operators are unitary, which means linear and invertible.

2.7 Born rule

Physical quantities written by different dimensions of physical units (meter, kilogram, second) represent number axes of mutually perpendicular coordinates, that is, the base of the vector space of a quantum system. It is a representation of vector space. Observables of a given *quantum system* span *vector space*. The individual vectors of such space are *quantum states*, and their *measurement* are the projections of vectors on the axes. The action of a unitary operator on a vector becomes an evolution of the quantum state.

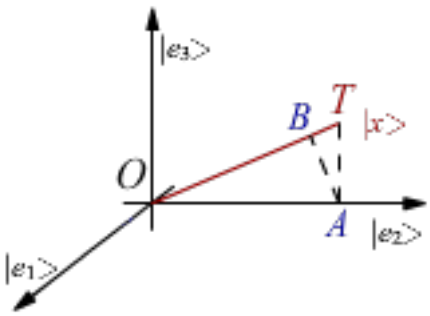
The quantum system is a material interpretation of the abstract Hilbert vector space and, therefore, it has some peculiarities. For example, in accordance with the principle of finiteness of matter, the atomization of each, but every, property of matter, so that the quantum state is always a set of particles, each with at most many material properties. The same implies the limited predictability of the physical behavior of the particles, that is, the principled inevitability of *coincidence* and then the need for probabilities. Consistently, the components of the state vector are some probability functions.

This was otherwise observed as early as 1926 when the *Born law*[25] was formulated for the probability of measurement (see [26]). In its simplest form, the law states that the density of the probability of finding a particle at a given place is proportional to the square of the amplitude of the particle of the wave function at that place. Varadarajan[26] is one of the first and best known to successfully explain why this probability equals exactly a square of amplitude and not another function, but instead of his we follow a consistent, slightly different one here (see [7]) and simpler explanations.

Consider the vector $|x\rangle$ represented by the longer OT in the image 2.3. The orthogonal projection of the point T on the axis $|e_2\rangle$ is the point A. *Dirac notation* and parallel measurement observable in the image we write

$$\overrightarrow{OA} = |e_2\rangle\langle e_2|x\rangle. \tag{2.104}$$

[25] Max Born (1882-1970), German mathematician and theoretical physicist.
[26] Veeravalli S. Varadarajan, born on May 1937, Indian mathematician.



Slika 2.3: Born probability.

Vectors representing different physical properties are not collinear (they are orthogonal) and their sum is not equal to the scalar sum of the intensities of the parts. Therefore, the quotient of the numbers $\langle e_2|x\rangle$ and $|x\rangle$ cannot represent the probability. To get the participation of individual coordinates in the total probability, we calculate the contribution of each component of the vector along the direction $|x\rangle$. We get

$$\overrightarrow{OB} = |x\rangle\langle x|\overrightarrow{OA} = |x\rangle\langle x|e_2\rangle\langle e_2|x\rangle = |x\rangle|\langle e_2|x\rangle|^2, \tag{2.105}$$

where we used the previous one (2.104). In total, the vector of the system is given by the sum of the contributions of the individual components along the direction $|x\rangle$, that is

$$\begin{aligned}|x\rangle &= |\langle e_1|x\rangle|^2|x\rangle + |\langle e_2|x\rangle|^2|x\rangle + \cdots + |\langle e_n|x\rangle|^n|x\rangle = \\ &= (|\langle e_1|x\rangle|^2 + |\langle e_2|x\rangle|^2 + \cdots + |\langle e_n|x\rangle|^2)|x\rangle = |x\rangle,\end{aligned}$$

because the sum of the probabilities of independent outcomes (the expression in parentheses) is one. That's why we have to use squares for probabilities

$$|\langle e_\mu|x\rangle|^2 = |\alpha_\mu|^2 = \alpha_\mu^*\alpha_\mu \tag{2.106}$$

and not some other function of the amplitude α_μ in representing the state vector.

The uncertainties of matter are not fictitious to us, they are real, we say objective because we cannot avoid them. The probabilities that arise from that are also real. Therefore, for material probabilities, the *likelihood principle* is, roughly speaking, the most likely thing to happen (see [8]). Because of these inevitable uncertainties that are realized by the laws of probability, figuratively speaking, I am here where I am and not there somewhere, because from my point of view, such a position is most likely at this time. You are there and not here because from your point of view this position is most likely. Therefore, the probability perception is relative to the individual particles. Each moves from its point of view to the most likely trajectories.

On the other hand, the realization of uncertainty generates *information*. Both uncertainty and information are material properties, and are therefore similar to other physical substance properties.

2.7.1 The cosines of angles

It is strange at first glance that in the representation of the quantum state $|x\rangle$ using the base states $|e_\mu\rangle$ in an otherwise common expression for vectors

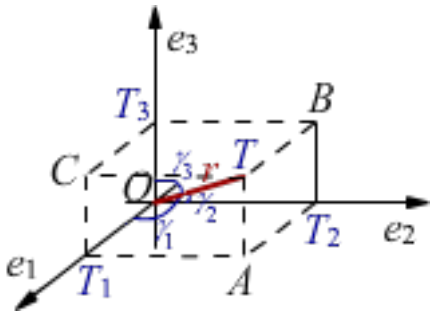
$$|x\rangle = \sum_{\mu=1}^{n} \alpha_\mu|e_\mu\rangle, \tag{2.107}$$

or by notation in the manner (2.104), the coefficients α_μ appear which by themselves are not probabilities, but probabilities are their squares $|\alpha_\mu|^2 = \alpha_\mu^* \alpha_\mu$. This is the meaning of the aforementioned *Born Rule*, just explained. We will now clarify this in another, more abstract, yet more familiar way.

In a rectangular coordinate system whose axes from the origin O are defined by the unit vectors e_1, e_2, e_3 in the figure 2.4, the point T is given. Its vertical projections on the plane Oe_1e_2, Oe_2e_3, and Oe_3e_1 are, respectively, the points A, B, and C. The projections of these points sequentially to the coordinate axes are the points T_1, T_2, and T_3. The angles that the vector $\overrightarrow{OT}$ closes with the coordinate axes are in the order γ_1, γ_2, and γ_3. We know from mathematics (analytical geometry) that

$$\cos^2\gamma_1 + \cos^2\gamma_2 + \cos^2\gamma_3 = 1, \tag{2.108}$$

and this is a confirmation of Born rule in the representation of (real) rectangular Descartes systems.



Slika 2.4: The rectangles in a rectangular system.

Example 2.7.1. *Let's prove (2.108).*

Proof. From the figure 2.4 we can see:

$$\overline{OT}^2 = \overline{OA}^2 + \overline{AT}^2, \quad \overline{OA}^2 = \overline{OT}_1^2 + \overline{AT}_1^2,$$

because the triangles OAT and OT_1A are right-angled. It is a right-angled triangle OT_1T. We have analogy with the other two axes. From the trigonometry of a right triangle it follows:

$$\cos^2\gamma_1 = \left(\frac{\overline{OT}_1}{\overline{OT}}\right)^2, \quad \cos^2\gamma_2 = \left(\frac{\overline{OT}_2}{\overline{OT}}\right)^2, \quad \cos^2\gamma_3 = \left(\frac{\overline{OT}_3}{\overline{OT}}\right)^2,$$

$$\cos^2\gamma_1 + \cos^2\gamma_2 + \cos^2\gamma_3 = \frac{\overline{OT}_1^2 + \overline{OT}_2^2 + \overline{OT}_3^2}{\overline{OT}^2} = \frac{\overline{OT}^2}{\overline{OT}^2} = 1,$$

and that needed to be proven. □

So the Born law, along with the "wonders" of quantum mechanics, is part of the larger picture, impeccably logical in the ways that mathematics can be. Quantum mechanics, on the other hand, is a material representation of that logic, which means that it has its own phenomena.

2.8 Tensor product

We are looking at some random event with two outcomes, probability $u_\mu, v_\nu \in [0,1]$, where the indices are $\mu, \nu \in \{1, 2, \ldots, n\}$ and conditions apply

$$\begin{cases} u_1 + u_2 + \cdots + u_n = 1, \\ v_1 + v_2 + \cdots + v_n = 1. \end{cases} \tag{2.109}$$

Shannon's information on such events are:

$$S_u = -\sum_{\mu=1}^{n} u_\mu \log_b u_\mu, \quad S_v = -\sum_{\nu=1}^{n} v_\nu \log_b v_\nu, \tag{2.110}$$

in some base ($b > 0$ and $b \neq 1$) logarithms, that is

$$\begin{cases} S_u = -u_1 \ln u_1 - u_2 \ln u_2 - \cdots - u_n \ln u_n, \\ S_v = -v_1 \ln v_1 - v_2 \ln v_2 - \cdots - v_n \ln v_n, \end{cases} \tag{2.111}$$

in natural logarithms, bases $e = 2.71828\ldots$, thus defining "nat" as a unit of information. We can write these events as vectors $\mathbf{u}$ and $\mathbf{v}$ in some n-dimensional coordinate system:

$$\mathbf{u} = \sum_{\mu=1}^{n} a_\mu \mathbf{e}_\mu, \quad \mathbf{v} = \sum_{\nu=1}^{n} b_\nu \mathbf{e}_\nu, \tag{2.112}$$

where $\mathbf{e}_\mu$ ($\mu = 1, 2, \ldots, n$) are unit coordinate vectors. If we allow the possibility that the coordinates are complex numbers ($a_\mu, b_\nu \in \mathbb{C}$) then $u_\mu = a_\mu^* a_\mu$ and $v_\nu = b_\nu^* b_\nu$, so the scalar (internal) products of these vectors apply:

$$\begin{cases} \langle \mathbf{u}|\mathbf{u}\rangle = \sum_\mu a_\mu^* a_\mu = \sum_\mu u_\mu = 1, \\ \langle \mathbf{v}|\mathbf{v}\rangle = \sum_\nu b_\nu^* b_\nu = \sum_\nu v_\nu = 1. \end{cases} \tag{2.113}$$

The asterisk (z^*) denotes the conjugation of a complex number (z). We look at the tensor product of these vectors (distributions) $\mathbf{u} \otimes \mathbf{v}$ and the information S of that product.

The tensor product of the events (2.112) is:

$$\mathbf{u} \otimes \mathbf{v} = (a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + \cdots + a_n\mathbf{e}_n) \otimes (b_1\mathbf{e}_1 + b_2\mathbf{e}_2 + \cdots + b_n\mathbf{e}_n) = \sum_{\mu,\nu=1}^{n} a_\mu b_\nu \, \mathbf{e}_\mu \otimes \mathbf{e}_\nu,$$

that is

$$\mathbf{u} \otimes \mathbf{v} = \sum_{\mu,\nu=1}^{n} a_\mu b_\nu \mathbf{e}_{\mu\nu}, \tag{2.114}$$

where are $\mathbf{e}_{\mu\nu} = \mathbf{e}_\mu \otimes \mathbf{e}_\nu$ unit vectors of coordinates of the tensor product of the system. Because of (2.113) then we have

$$\langle \mathbf{u} \otimes \mathbf{v}|\mathbf{u} \otimes \mathbf{v}\rangle = \sum_{\mu,\nu=1}^{n} u_\mu u_\nu = 1. \tag{2.115}$$

Namely:

$$\langle \mathbf{u} \otimes \mathbf{v}|\mathbf{u} \otimes \mathbf{v}\rangle = \sum_{\mu,\nu=1}^{n} (a_\mu^* b_\nu^*)(a_\mu b_\nu) = \sum_{\mu,\nu=1}^{n} (a_\mu^* a_\nu)(b_\mu^* b_\nu) = \sum_{\mu,\nu=1}^{n} u_\mu v_\nu =$$

$$= \sum_{\mu=1}^{n} \sum_{\nu=1}^{n} u_\mu v_\nu = \sum_{\mu=1}^{n} \left(\sum_{\nu=1}^{n} v_\nu \right) u_\mu = \sum_{\mu=1}^{n} u_\mu = 1,$$

thereby proving (2.115).

2.8.1 Adding information

Like throwing two coins each with an outcome of Heads or Tails where we get four results HH, HT, TH and TT, in the more general case of the distribution of unequal probabilities of two outcomes ($n = 2$) we have the vector tensor product (2.112) that we now write:

$$\begin{cases} \mathbf{r}_a = p_a\,\mathbf{e}_1 + q_a\,\mathbf{e}_2, \\ \mathbf{r}_b = p_b\,\mathbf{e}_1 + q_b\,\mathbf{e}_2, \end{cases} \tag{2.116}$$

where $p_a, p_b, q_a, q_b \in (0,1)$, $p_a + q_a = 1$, $p_b + q_b = 1$.The tensor product of these vectors is the vector:

$$\mathbf{r}_a \otimes \mathbf{r}_b = (p_a\mathbf{e}_1 + q_a\mathbf{e}_2) \otimes (p_b\mathbf{e}_1 + q_b\mathbf{e}_2) =$$

$$= p_a p_b\,\mathbf{e}_1 \otimes \mathbf{e}_1 + p_a q_b\,\mathbf{e}_1 \otimes \mathbf{e}_2 + q_a p_b\,\mathbf{e}_2 \otimes \mathbf{e}_1 + q_a q_b\,\mathbf{e}_2 \otimes \mathbf{e}_2$$

$$= p_a p_b\,\mathbf{e}_{11} + p_a q_b\,\mathbf{e}_{12} + q_a p_b\,\mathbf{e}_{21} + q_a q_b\,\mathbf{e}_{22},$$

where are $\mathbf{e}_{\mu\nu}$ unit vectors 4-dim coordinates of the tensor product 2-dim space. Of course, we can also use the previous labels, as in the following lemma.

Lemma 2.8.1. *For the information of the 2-dimensional tensor events is valid*

$$S = S_u + S_v, \tag{2.117}$$

for independent events.

Proof. The information of such a vector product is:

$$S = -u_1v_1 \ln u_1v_1 - u_1v_2 \ln u_1v_2 - u_2v_1 \ln u_2v_1 - u_2v_2 \ln u_2v_2 =$$

$$= -(u_1v_1 + u_1v_2)\ln u_1 - (u_1v_1 + u_2v_1)\ln v_1 - (u_2v_1 + u_2v_2)\ln u_2 - (u_1v_2 + u_2v_2)\ln v_2$$

$$= -u_1(v_1 + v_2)\ln u_1 - v_1(u_1 + u_2)\ln v_1 - u_2(v_1 + v_2)\ln u_2 - v_2(u_1 + u_2)\ln v_2$$

$$= -u_1 \ln u_1 - v_1 \ln v_1 - u_2 \ln u_2 - v_2 \ln v_2$$

$$= S_u + S_v,$$

so (2.117) is proved. □

Let's look at this with a little logarithmic calculating. From (2.117) we obtain:

$$-\ln(u_1^{u_1}v_1^{v_1}u_2^{u_2}v_2^{v_2}) = -\ln(u_1^{u_1}v_1^{v_1}) - \ln(u_2^{u_2}v_2^{v_2}), \tag{2.118}$$

which is true. This is then easily generalized into equality

$$-\ln(u_1^{u_1}v_1^{v_1}u_2^{u_2}v_2^{v_2}\ldots u_n^{u_n}v_n^{v_n}) = -\ln(u_1^{u_1}v_1^{v_1}) - \ln(u_2^{u_2}v_2^{v_2}) - \cdots - \ln(u_n^{u_n}v_n^{v_n}), \tag{2.119}$$

for each natural number $n = 1, 2, 3, \ldots$, and hence the following lemma.

Lemma 2.8.2. *If for a given natural number $n \in \mathbb{N}$ and each $k = 1, 2, \ldots, n$ is valid $p_k, q_k \in [0,1]$, $p_k + q_k = 1$ and $S_k = -p_k \ln p_k - q_k \ln q_k$, then is $S = S_1 + \cdots + S_n$ Shannon information of the tensor product of all vectors $\mathbf{r}_k = p_k\,\mathbf{e}_1 + q_k\,\mathbf{e}_2$ for $k = 1, 2, \ldots, n$ orderly.*

Finally, applying these two lemmas and their modes of proof, can be proved the 2.8.3 theorem.

Theorem 2.8.3. *If (independent) probability distributions (2.109) of information (2.110) are given by vectors (2.112), then the information of their tensor product (2.114) is equal to the sum of the given information, $S = S_a + S_b$.*

Proof. The tensor product information is:

$$S = -\sum_{\mu,\nu=1}^{n} u_\mu v_\nu \ln u_\mu v_\nu = \sum_{\mu=1}^{n}\sum_{\nu=1}^{n} u_\mu v_\nu(-\ln u_\mu - \ln v_\nu) =$$

$$= \sum_{\nu=1}^{n} v_\nu \left(\sum_{\mu=1}^{n} -u_\mu \ln u_\mu\right) + \sum_{\mu=1}^{n} u_\mu \left(\sum_{\nu=1}^{n} -v_\nu \ln v_\nu\right)$$

$$= \sum_{\nu=1}^{n} v_\nu S_a + \sum_{\mu=1}^{n} u_\mu S_b = S_a + S_b,$$

and thus the theorem is proved. □

2.8.2 Operator multiplication

Let's look at the product of binomials

$$(\hat{A} + \hat{B})(\hat{A} + \hat{B}) = \hat{A}^2 + (\hat{A}\hat{B} + \hat{B}\hat{A}) + \hat{B}^2, \tag{2.120}$$

where $\hat{A}$ and $\hat{B}$ are linear operators. Quantum states (say $|\psi\rangle$) are known to be representations of the Hilbert algebra vectors, and that evolutions are unitary operators (say $\hat{A}$). With such, its eigen (own) equation

$$\hat{A}|\psi\rangle = a|\psi\rangle, \tag{2.121}$$

gives an eigenvalue indicating the probability (a^*a) that the observables will be realized. It's familiar so I don't last (see [4]).

What might be new here is the observation that the sum of the information is derived from the two actions like

$$(\hat{A} + \hat{B})|\psi\rangle = (a + b)|\psi\rangle, \tag{2.122}$$

smaller than the coupled information of the square (2.120) when

$$\hat{A}\hat{B} + \hat{B}\hat{A} = 0, \tag{2.123}$$

that is, when the operators are *anti-commutative*. This is simply because

$$-u^2 \ln u^2 - v^2 \ln v^2 < 2(-u \ln u - v \ln v), \tag{2.124}$$

for proper distribution. This (see below) means that *fermions* are deficient in information, that *Pauli's exclusion principle* applies where there is a lack of information. An example of this is an atom.

The lack of fermion information and this kind of attachment related to Pauli's exclusion principle should be distinguished from the lack of information due to saturation, which you can follow in the various discrete distributions detailed in my book Physical Information (see [2]). On the other hand, there is a different reduction in information such as *quantum entanglement*.

2.9 Qubit

In closed quantum systems, some physical quantities do not change their value. They are subject to *conservation laws* like energy conservation: "energy cannot originate from nothing or disappear into nothing, but can only change from one form to another". By closed physical systems we mean isolated from the action of external forces and those in which mass, energy, momentum, spin, and even information are conserved. If their total is constant over time they are *canned systems*.

From [8] we know that force changes probabilities and therefore information, but that the total amount of uncertainty and information produced is constant under the action of forces. However, in the absence of (external) forces, there is no change in either the amount of uncertainty or the information, so in canned systems (in the absence of external forces) the *conservation of information* law applies.

Therefore, similar to classical technical feedback *gates* or *circuits*, the spontaneous development of the quantum system (evolution of the quantum state) does not lose information. As the system evolves, information is constantly stored (in the past). However, unlike technical gadgets, we can't just go back in time, so we need a new name for "quantum door" to distinguish them. We also have terms *switch* or *valves* (in my texts) for simple technical circuits, but here we will use the old names, the circuits and the gates, implying reversibility, because we do not deal with others.

Of course, for a mathematician, the choice of such names is arbitrary, but it is also about physics and technology. Unlike the technical device we make and use, we pin the quantum state and then adjust it if necessary. It develops only or is influenced by external forces in a way that we, for the time being, can barely control. Unlike classic computers that humans did not find in nature, but made them to have, only recently (since the 20th century) have we noticed that quantum computers are all around us, and the question is whether and when we will be able to use. This is one of the reasons for using different terms.

Hadamard's[27] *gate* works on one *qubit* (quantum bit - unit of quantum information). The state of the qubit $|\psi\rangle$ is a linear *superposition* of the basic states of quantum mechanics $|1\rangle$ and $|0\rangle$, say

$$|1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \tag{2.125}$$

which in the macro-world match the outcome of a coin toss:

$$|\psi\rangle = z_0|0\rangle + z_1|1\rangle. \tag{2.126}$$

The complex numbers $z_0, z_1 \in \mathbb{C}$ are the amplitudes of *Born* probabilities. The sum of the squares of the modules of these numbers is one

$$|z_0|^2 + |z_1|^2 = 1, \tag{2.127}$$

when the realization of at least one of the basic states is a certain event.

When measuring a qubit on a standard basis, the outcome $|0\rangle$ has a probability $|z_0|^2$, and the outcome $|1\rangle$ a probability $|z_1|^2$. The sum of the squares of cosines and sines of an angle is a unit, so (2.126) can be written as a rotation, by substituting $z_0 = \cos\frac{\theta}{2}$ and $z_1 = e^{i\phi}\sin\frac{\theta}{2}$. The special rotation of the qubit is the Hadamard transform, which we now call the Hadamard gate.

[27] Jacques Hadamard (1865-1963), French mathematician.

Using *Dirac Notation*, we define Hadamard's gate as:

$$|0\rangle \to \frac{|0\rangle + |1\rangle}{\sqrt{2}}, \quad |1\rangle \to \frac{|0\rangle - |1\rangle}{\sqrt{2}}, \tag{2.128}$$

so we continue with (2.126):

$$|\psi\rangle = z_0|0\rangle + z_1|1\rangle \to z_0\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\right) + z_1\left(\frac{|0\rangle - |1\rangle}{\sqrt{2}}\right),$$

which after algebraic updating becomes

$$|\psi\rangle = \frac{z_0 + z_1}{\sqrt{2}}|0\rangle + \frac{z_0 - z_1}{\sqrt{2}}|1\rangle. \tag{2.129}$$

We are writing an external product of this superposition

$$\frac{|0\rangle + |1\rangle}{\sqrt{2}}\langle 0| + \frac{|0\rangle - |1\rangle}{\sqrt{2}}\langle 1|,$$

and this corresponds to the *Hadamard matrix*

$$\mathbf{H} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \tag{2.130}$$

in the base $|0\rangle$, $|1\rangle$. It is obvious that consecutive conjugation and transposition does not change the Hadamard matrix, $\mathbf{H}^\dagger = \mathbf{H}$, which in itself means that this matrix is symmetric and its square is an identical, unit matrix, $\mathbf{H}^2 = \mathbf{I}$. These few pages are just announcements of the following topics.

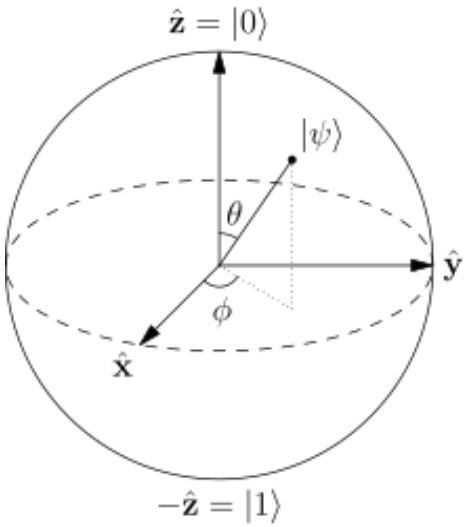
2.9.1 Many worlds

The aforementioned quantum rotations of the qubit, from which we derived Hadamard's, are on the *Bloch sphere* shown in the figure 2.5. In quantum mechanics, the Bloch sphere is a geometric representation of two-level quantum states, such as qubits.

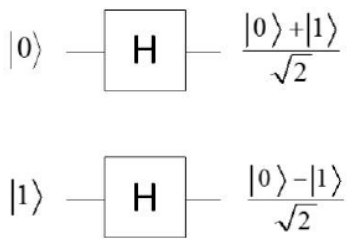
It is a sphere of unit radius in *Hilbert space*, that is projective Hilbert space, with opposite points corresponding to a pair of mutually perpendicular state vectors. The north and south poles of the Bloch sphere are usually chosen to correspond to the basis vectors $|0\rangle$ and $|1\rangle$ which in turn corresponds to the upper and lower states of the spin electrons. The points on the surface of the sphere correspond to *pure states* (one of which will surely occur), while the interior of the sphere corresponds to *mixed states* (whose sum of probabilities is less than one). The Bloch sphere can be extended to a n-dimensional quantum system, but it then ceases to be a model for qubit and its visual benefit is smaller.

The symbol and operation of the Hadamard gate is shown in the figure 2.6. We note that it inverts the declared states into two equally probable unexplained states and vice versa. Because of the generality of this gate, it is a general observation about the evolution of quantum states. With the principle of probability, the *axiom of objective coincidence* in [8], this observation leads to unusual conclusions about the nature of the universe.

For example, if by throwing a coin, we make the decision to choose the path to continue our life and if dropping "heads" or "tails" is a kind of objective coincidence, then no matter how the coin falls – the laws of physics will not change. However, I will not live in the



Slika 2.5: Bloch sphere.



Slika 2.6: Hadamard gate.

same physical reality after the election of the "head" as after the election of the "tail". Let's call them parallel realities in the *multiverse*, names from speculative physics with roots in ancient Greek mythology. In 1957, *Everett* on the similar based his interpretation of quantum mechanics, called *many-worlds*.

After his now celebrated theory of multiple universes met scorn, Hugh Everett abandoned the world of academic physics, and turned to top secret military research and led a tragic private life[28] so his basic idea remains unfinished. For example, the question of how our consciousness travels "packed" into 4-D space-time[29] while the wider environment is at least 6-D? That is why this book was inevitable with its equation of information with physical action, so that we could say that without change of time there is no information. The product of the uncertainty (change) of energy and time is a quantum of action as well as it is the information, so when there is no penetration along the time axis then there is no information in that of many-worlds.

That there is no significant direct physical communication between the two realities also follows from the law of conservations. Hadamard's gates will translate uncertainty into certainty, into only one of ME, but will do the opposite in the second successive actions; it will bring my parallel ME back into one of my previous uncertainty options. That's right from my point of view. From the point of view of the observer (particle) which time goes

[28]Quote from: `https://www.scientificamerican.com/article/hugh-everett-biography/`.
[29]see "1.12 Free will" in [2].

back, the observation of these events is symmetrical. To one observer, the Hadamard gate translates uncertainties into information, and to another he purchases such realizations from parallel realities, then as uncertainties, translating them back into information.

For one of the two observers of opposite flows of time the realization of uncertainty in information occurs, while for the other the opposite happens. To others, information and uncertainty have alternated roles.

Let's also look at the known physics and the familiar tasks related to the Hadamard gate. Many quantum algorithms use the Hadamard transform as an initial step, when mapping $n = 1, 2, 3, \ldots$ qubits can be done one at a time. Hadamard's matrix can also be regarded as Fourier[30] transformation on a two-element additive group (Module 2).

2.9.2 Examples of superpositions

Example 2.9.1. *Apply Hadamard matrices to basic vectors.*

Solution. We assume that:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \alpha|0\rangle + \beta|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}.$$

Matrix multiplication gives:

$$\mathbf{H}|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{|0\rangle + |1\rangle}{\sqrt{2}},$$

$$\mathbf{H}|1\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{|0\rangle - |1\rangle}{\sqrt{2}}.$$

□

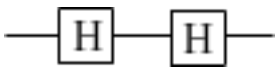
Example 2.9.2. *Successive application of two Hadamard gates to basic vectors.*

Solution. We have successive transformations (figure 2.7):

$$|0\rangle \to \frac{|0\rangle + |1\rangle}{\sqrt{2}} \to \frac{1}{\sqrt{2}} \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} + \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) = |0\rangle,$$

$$|1\rangle \to \frac{|0\rangle - |1\rangle}{\sqrt{2}} \to \frac{1}{\sqrt{2}} \left(\frac{|0\rangle + |1\rangle}{\sqrt{2}} - \frac{|0\rangle - |1\rangle}{\sqrt{2}} \right) = |1\rangle.$$

In other words, the square of the Hadamard matrix is a unit matrix. □



Slika 2.7: Two Hadamard gates.

It is known that the so-called first *Pauli*[31] matrix, σ_x, is representation of negation in quantum circuits:

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_x|1\rangle = |0\rangle, \quad \sigma_x|0\rangle = |1\rangle. \tag{2.131}$$

[30] Joseph Fourier (1768-1830), French mathematician and physicist.
[31] Wolfgang Pauli (1900-1958), Austrian-Swiss-American theoretical physicist.

The following but slightly more complex examples are similar; they are for the level of students of quantum mechanics or information technology.

Example 2.9.3. *Write a matrix for rotation about the y-axis Bloch sphere for an extended angle (π radian). It maps $|0\rangle \to i|1\rangle$ and $|1\rangle \to -i|0\rangle$.*

Solution. This is Pauli's second matrix:

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \tag{2.132}$$

By matrix multiplication we get:

$$\sigma_y|0\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ i \end{pmatrix} = i|1\rangle, \quad \sigma_y|1\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -i \\ 0 \end{pmatrix} = -i|0\rangle.$$

it is also $\sigma_y^2 = I$. □

Example 2.9.4. *Write a rotation matrix about z-axis Bloch sphere for an extended angle, 180°. It leaves the base state $|0\rangle$ unchanged, and maps $|1\rangle$ to $-|1\rangle$.*

Solution. This is Pauli's third matrix:

$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.133}$$

By matrix multiplication we get:

$$\sigma_z|0\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle, \quad \sigma_z|1\rangle = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} = -|1\rangle.$$

It's again $\sigma_z^2 = I$. □

Pauli matrices are also defined so that their square is a unit matrix, $\sigma^2 = I$. On the other hand, the Hadamard matrix is $H = \frac{1}{\sqrt{2}}(\sigma_x + \sigma_z)$. The *involutory matrix* is one that is inverse to itself. In other words, the multiplication of the matrix A is involutority (self-inversion) if and only if $A^2 = I$. Involutory matrices are the square roots of unit matrices.

Example 2.9.5 (Involutory matrix)**.** *Let $x \in \mathbb{R}$, and σ is an involutory matrix, i.e. such that its square is a unit matrix, $\sigma^2 = I$. Show that it is then $e^{ix\hat{\sigma}} = I\cos x + i\hat{\sigma}\sin x$.*

Solution. We use Maclaurin, Taylor's developments in series:

$$\begin{aligned} e^{ix\hat{\sigma}} &= \sum_{k=0}^{\infty} \frac{(ix\hat{\sigma})^k}{k!} = \sum_{k=0}^{\infty} \frac{(ix\hat{\sigma})^{2k}}{(2k)!} + \sum_{k=0}^{\infty} \frac{(ix\hat{\sigma})^{2k+1}}{(2k+1)!} = \\ &= \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}\,\hat{I} + i\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}\,\hat{\sigma} \\ &= (\cos x)\,\hat{I} + i(\sin x)\,\hat{\sigma}. \end{aligned}$$

□

Example 2.9.6. *Express the Hadamard matrix using Pauli products.*

Solution. From the previous example it is:

$$\exp(i\pi\sigma_x/4) = (I + i\sigma_x)/\sqrt{2}, \quad \exp(i\pi\sigma_z/4) = (I + i\sigma_z)/\sqrt{2}.$$

То су ротације σ_x и σ_z за угао $\pi/4$. Отуда:

$$\frac{(I + i\sigma_x)}{\sqrt{2}}\frac{(I + i\sigma_z)}{\sqrt{2}}\frac{(I + i\sigma_x)}{\sqrt{2}} = \frac{1}{2\sqrt{2}}(I + i(\sigma_x + \sigma_z) - \sigma_x\sigma_z)(I + i\sigma_x) =$$

$$= \frac{1}{2\sqrt{2}}(I + i(\sigma_x + \sigma_z) - \sigma_x\sigma_z + i\sigma_x - I - \sigma_z\sigma_x - i\sigma_x\sigma_z\sigma_x)$$

$$= \frac{i}{\sqrt{2}}(\sigma_x + \sigma_z) = iH,$$

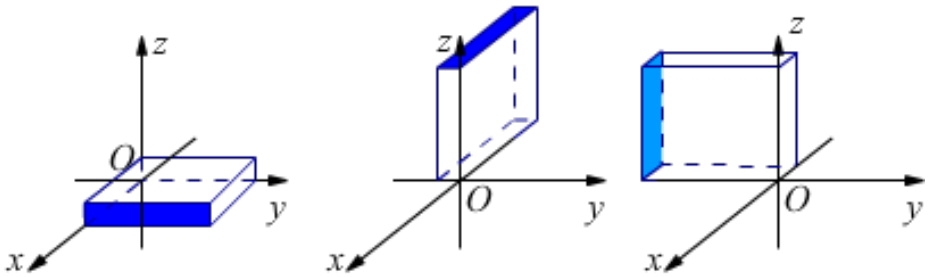
because $\sigma_x\sigma_z + \sigma_z\sigma_x = 0$ следи $\sigma_z\sigma_z\sigma_x = -\sigma_z$. Hence

$$H = e^{-i\pi/2}e^{i\pi\sigma_x/4}e^{i\pi\sigma_z/4}e^{i\pi\sigma_x/4}.$$

□

2.10 Uncertainty principle

We know that dependent processes of quantum mechanics are represented by noncommutative operators. Let ρ_x and ρ_y be the rotations about x and y-axes for 90° of the rectangular Descartes system $Oxyz$ in the figure 2.8, and K a square represented on the first picture to the left. Then K_{yx} is the square in the image in the middle, obtained by the composition $\rho_y \circ \rho_x : K \to K_{yx}$, and in the same image on right is the square $\rho_x \circ \rho_y : K \to K_{xy}$. Obviously, $K_{yx} \neq K_{xy}$, or $\rho_y \circ \rho_x \neq \rho_x \circ \rho_y$. Thus, the compositions of rotations in 3-D are not commutative.



Slika 2.8: Rotations of the square around x and y axes.

Another example is the *perception information* (2.2) where the vectors of "intelligence" and "hierarchy" are aligned with each other so that their scalar product is as small as possible. The third example mentioned here (2.123) is anti-commutativity.

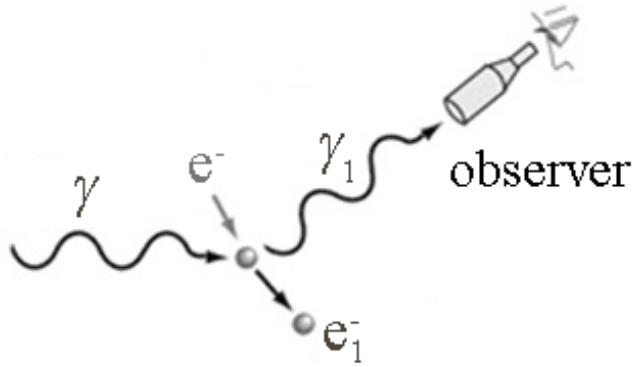
Uncertainty of electron

A special situation arises by measuring the momentum and position of the electron e^- by colliding with the photon γ shown in the figure 2.9. It is one of the first descriptions after discovering Heisenberg's *uncertainty* [25] and the founding of quantum mechanics.

In these initial considerations of the experiments it was observed that there were limits to what we could see about the electron. From the "elementary formula of the Compton effect", Heisenberg in 1927 *estimated* that the product of the uncertainty of the momentum measurement Δp of a electron and its position Δx must be of the order

$$\Delta p \Delta x \sim h, \tag{2.134}$$

where h is Plank's constant. The electron momentum is p and its position is x. The position of the photon is smeared along the wavelength λ and its momentum is the quotient h/λ. The observation of electrons after a collision with a photon becomes smeared in $\Delta p = h/\lambda$ and $\Delta x = \lambda$, which, by replacing it with the previous one, gives $\Delta p = h/\Delta x$, so $\Delta p \Delta x = h$, and this is the Heisenberg's estimate.



Slika 2.9: Uncertainty of electron.

The same estimate can be obtained using particles with mass m and velocity v, using the *Louis de Broglie* expression for wavelength $\lambda = h/p$. Then $\Delta p = mv$ and $\Delta x = h/mv$, so by substitution we again get $\Delta p \Delta x = h$.

From these methods, we find that the uncertainties of momentum and position measurements come from scattering within a wavelength, which does not give us the exact position of what we are measuring, and therefore of the measuring. So it may seem to us that it has little to do with real coincidence, but rather that it is a consequence of only our "bad" tools. But as Louis de Broglie's law on the wave nature of a substance applies to every kind of matter, we conclude that there are no "better" tools. So there is nothing in this universe that can challenge Heisenberg's estimate (2.134), so it remains to be seen whether it agrees with the laws of probability.

In 1927 Kennard[32] (see [24]) proved this to be true, and in the following year Weyl[33] (see [23]) discover the formula for the dispersions of momentum σ_p and the position σ_x of particles:

$$\sigma_p \sigma_x \geq \frac{\hbar}{2}, \tag{2.135}$$

where $\hbar = h/(2\pi)$ is reduced Planck's constant.

[32]Earle Hesse Kennard (1885-1968), theoretical physicist and professor at Cornell University.
[33]Hermann Weyl (1885-1955), German mathematician, theoretical physicist and philosopher.

2.10.1 The uncertainties of dispersions

Let's look at this in more detail. The average or *mean* of numbers $x_1, x_2, \ldots, x_n$, or $p_1, p_2, \ldots, p_n$ is often indicated by a line above, so we write:

$$\bar{x} = \frac{x_1 + x_2 + \cdots + x_n}{n}, \quad \bar{p} = \frac{p_1 + p_2 + \cdots + p_n}{n}, \tag{2.136}$$

unlike Dirac's *notation*, which would write $\langle x \rangle$ and $\langle p \rangle$ with the same mean. The interval of error or uncertainty in determining these numbers is:

$$\Delta x = x - \bar{x}, \quad \Delta p = p - \bar{p}. \tag{2.137}$$

The mean each of these intervals is zero:

$$\overline{\Delta x} = \overline{x - \bar{x}} = \bar{x} - \bar{x} = 0, \quad \overline{\Delta p} = \overline{p - \bar{p}} = \bar{p} - \bar{p} = 0, \tag{2.138}$$

which is obvious. The mean square deviation defines *dispersion* σ:

$$\begin{cases} \overline{(\Delta x)^2} = [(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \cdots + (x_n - \bar{x})^2]/n \\ \overline{(\Delta p)^2} = [(p_1 - \bar{p})^2 + (p_2 - \bar{p})^2 + \cdots + (p_n - \bar{p})^2]/n, \end{cases} \tag{2.139}$$

for which is here $\sigma^2(x) = \overline{(\Delta x)^2}$, $\sigma^2(p) = \overline{(\Delta p)^2}$, so for the measurement errors we take

$$\sigma(x) = \sqrt{\overline{(\Delta x)^2}}, \quad \sigma(p) = \sqrt{\overline{(\Delta p)^2}}. \tag{2.140}$$

This is how Heisenberg's uncertainty relations become

$$\sigma(x) \cdot \sigma(p) \geq \frac{\hbar}{2}, \tag{2.141}$$

in accordance with (2.135). Only abscissa is implied. This result was also obtained by Heisenberg (see [22]), in a slightly different way.

2.10.2 Mean square deviations

The mean square deviation is taken as:

$$\overline{(\Delta x)^2} = \overline{(x - \bar{x})^2} = \overline{x^2} - 2\overline{x}\overline{x} + \bar{x}^2 = \overline{x^2} - \bar{x}^2, \quad \overline{(\Delta p)^2} = \overline{(p - \bar{p})^2} = \overline{p^2} - \bar{p}^2,$$

when the coordinate origin is the point $\bar{x}$ then $\bar{p} = 0$, so we have:

$$\overline{(\Delta x)^2} = \overline{x^2}, \quad \overline{(\Delta p)^2} = \overline{p^2}. \tag{2.142}$$

For the mean values in quantum mechanics we obtain:

$$\begin{cases} \overline{x^2} = \int_{-\infty}^{\infty} \psi^*(x) x^2 \psi(x)\, dx, \\ \overline{p^2} = \int_{-\infty}^{\infty} \psi^*(x) p^2 \psi(x)\, dx = -\hbar^2 \int_{-\infty}^{\infty} \psi^*(x) \frac{d^2\psi(x)}{dx^2}\, dx. \end{cases} \tag{2.143}$$

These are the original values obtained by Heisenberg and Weil, followed by Pauli and others. From the lower integral we obtain:

$$\int_{-\infty}^{\infty} \psi^* \frac{d^2\psi}{dx^2}\, dx = \int_{-\infty}^{\infty} \psi^*\, d\left(\frac{d\psi}{dx}\right) = \psi^* \frac{d\psi}{dx}\bigg|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} \frac{d\psi}{dx}\, d\psi^* =$$

$$= -\int_{-\infty}^{\infty} \frac{d\psi}{dx}\frac{d\psi^*}{dx}\,dx = -\int_{-\infty}^{\infty} \left|\frac{d\psi}{dx}\right|^2 dx,$$

and hence

$$\overline{p^2} = \hbar^2 \int_{-\infty}^{\infty} \left|\frac{d\psi}{dx}\right|^2 dx. \tag{2.144}$$

Heisenberg further continues with the obvious inequality:

$$\left|\frac{x}{2\overline{(\Delta x)^2}}\psi(x) + \frac{d\psi(x)}{dx}\right|^2 \geq 0,$$

$$\frac{x^2|\psi|^2}{4[\overline{(\Delta x)^2}]^2} + \frac{x\psi}{\overline{(\Delta x)^2}} \cdot \frac{d\psi}{dx} + \left(\frac{d\psi}{dx}\right)^2 \geq 0,$$

$$\left|\frac{d\psi}{dx}\right|^2 \geq -\frac{x\psi}{\overline{(\Delta x)^2}} \cdot \frac{d\psi}{dx} - \frac{x^2|\psi|^2}{4[\overline{(\Delta x)^2}]^2}.$$

But because:

$$\frac{d}{dx}\left[\frac{x|\psi|^2}{2\overline{(\Delta x)^2}}\right] = \frac{x\psi}{\overline{(\Delta x)^2}} \cdot \frac{d\psi}{dx} + \frac{|\psi|^2}{2\overline{(\Delta x)^2}},$$

$$-\frac{x\psi}{\overline{(\Delta x)^2}} \cdot \frac{d\psi}{dx} = -\frac{d}{dx}\left[\frac{x|\psi|^2}{2\overline{(\Delta x)^2}}\right] + \frac{|\psi|^2}{2\overline{(\Delta x)^2}},$$

by substitution we get:

$$\left|\frac{d\psi}{dx}\right|^2 \geq \frac{|\psi|^2}{2\overline{(\Delta x)^2}} - \frac{d}{dx}\left[\frac{x|\psi|^2}{2\overline{(\Delta x)^2}}\right] - \frac{x^2}{4}\frac{|\psi|^2}{[\overline{(\Delta x)^2}]^2}.$$

Multiply by $\hbar^2$ and integrate

$$\overline{p^2} = \overline{(\Delta p)^2} \geq \frac{\hbar^2}{4} \cdot \frac{1}{\overline{(\Delta x)^2}}\left[2\int_{-\infty}^{\infty} |\psi|^2\,dx - \int_{-\infty}^{\infty} \frac{x^2|\psi|^2 dx}{[\overline{(\Delta x)^2}]^2}\right] - \left.\frac{x|\psi|^2}{2\overline{(\Delta x)^2}}\right|_{-\infty}^{\infty}.$$

Next, from

$$\int_{-\infty}^{\infty} x^2|\psi|^2\,dx = \overline{x^2} = \overline{(\Delta x)^2},$$

we get

$$\overline{p^2} \geq \frac{\hbar^2}{4} \cdot \frac{1}{\overline{(\Delta x)^2}},$$

hence

$$\sqrt{\overline{(\Delta x)^2}} \cdot \sqrt{\overline{(\Delta p)^2}} \geq \frac{\hbar}{2}, \tag{2.145}$$

with marks equivalent to (2.140). The same can be written in the form

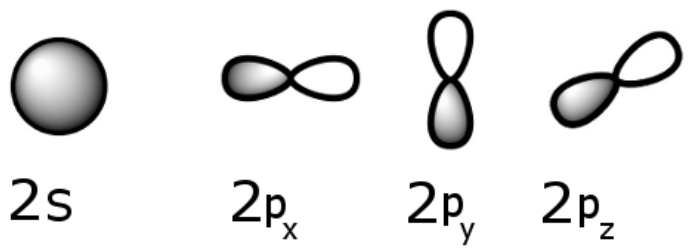
$$\Delta x \cdot \Delta p \geq \frac{\hbar}{2}, \tag{2.146}$$

which is a bit imprecise the previous expression.

2.10.3 Application in chemistry

Heisenberg's uncertainty principle quickly found application in chemistry. Orbits filled with electrons cannot be drawn accurately. These are obtained from the probabilities of electron distribution. At best, the electron's orbit can be estimated with a 90 % chance that the electron is in a given volume at a given time. The rest of the time it is somewhere else. The figure 2.10 are typical orbits.

Electrons are those parts that make an atom an atom, and the orbits of atoms are where the electrons are most likely to be located. The very word "orbit" came with Bohr Model of the Atom, in which electrons move around the nucleus similar to the planets around the sun. The first sphere, 1s, is closest to the nucleus of atoms with a maximum of two electrons having the lowest energy level. For helium (He) with two electrons, the first orbit is sufficient. However, lithium (Li) has three electrons and the third electron must go into the next sphere, the $2s$ orbit, to a higher energy level. Beryllium (Be) has two electrons in the first sphere ($1s$) and two electrons in the second sphere ($2s$).



Slika 2.10: Orbit electrons in an atom.

Boron (B) having five electrons begins to fill the third orbit. Its three orbits are perpendicular to each other and are denoted by 2p. The orbit, extending like the 3-D eights or balloons along the abscissa, is denoted by $2p_x$. Analogous, the $2p_y$ and $2p_z$, we denote orbits along ordinate and applicate. The shape of the orbit is determined by the likely positions of the electrons. An atom with even more electrons has both spherical and vertical orbits. Thus neon (Ne) has orbits $1s^2\ 2s^2\ 2p^6$, because it has ten electrons (2 + 2 + 6).

The further spherical orbit, 3s, takes on the next two electrons of even higher energies. Thus sodium (Na) may have 11 electrons, two each in the first two spherical orbits (1s and 2s), two each in vertical orbits ($2p_x$, $2p_y$ and $2p_z$) and one in the 3s spherical orbit. The atom can then have five d-orbits, each with a maximum of two electrons. The shape of each orbit is still such that the probability of finding electrons in it is about 90 %. Behind these are seven f-orbits, in which up to two electrons can also be found.

2.10.4 Avoiding uncertainty

The explanation of the orbits of atoms is one of the most important confirmations of Heisenberg's uncertainty principle. There are almost no serious doubts about this principle in physics today, but there are more and more accurate measurements that are being successfully avoided.

The recent work of the Institute of Photons (see [6]) has successfully directed the spin of electrons and nuclei by special techniques similar to the gyroscopic effect of atoms to obtain information on the position of the body of an atom and the principle of uncertainty is avoided. The trick comes from the observation that the spin has not only one but two

angles of orientation, one direction north-east-south-west and the other with elevation above the horizon.

The team of researchers showed how to put almost all uncertainty in an angle that is not measured by an instrument. That way, they still cling to Heisenberg's demand for uncertainty, but conceal the uncertainty that doesn't suit them. The result is the measurement of angular amplitude with unprecedented precision, which is not impeded by quantum uncertainty.

The participant of this experiment, prof. Mitchell says the uncertainty principle is very frustrating for scientists: "We would know everything, but Heisenberg says it is not possible. However, we did find a way to find out all that matters to us. As in the Rolling Stones song: 'You can't always get what you want / But if you try sometimes / Well, you might find / You get what you need'."

2.10.5 Calculation examples

Example 2.10.1. *Derive the indeterminacy relations (2.145) starting from the inequality*

$$\int_{-\infty}^{\infty} \left| \lambda x \psi + \frac{d\psi}{dx} \right|^2 \geq 0,$$

as obvious, so treat it further as a square inequality by λ.

Solution. We calculate orderly:

$$\int_{-\infty}^{\infty} \left(\lambda x \psi^* + \frac{d\psi^*}{dx} \right) \left(\lambda x \psi + \frac{d\psi}{dx} \right) \geq 0,$$

$$\int_{-\infty}^{\infty} \lambda^2 x^2 \psi^* \psi \, dx + \int_{-\infty}^{\infty} \lambda x \left(\psi^* \frac{d\psi}{dx} + \psi \frac{d\psi^*}{dx} \right) dx + \int_{-\infty}^{\infty} \frac{d\psi^*}{dx} \cdot \frac{d\psi}{dx} \, dx \geq 0,$$

$$\lambda^2 \overline{x^2} + \lambda x (\psi^* \psi) \Big|_{-\infty}^{\infty} - \lambda \int_{-\infty}^{\infty} \psi^* \psi \, dx + \frac{\overline{p^2}}{\hbar^2} \geq 0,$$

$$\overline{(\Delta x)^2} \lambda^2 - \lambda + \frac{\overline{(\Delta p)^2}}{\hbar^2} \geq 0.$$

For the inequality to be true, the quadratic equation discriminant holds:

$$1 - 4 \cdot \overline{(\Delta x)^2} \cdot \frac{\overline{(\Delta p)^2}}{\hbar^2} \leq 0,$$

$$\overline{(\Delta x)^2} \cdot \overline{(\Delta p)^2} \geq \frac{\hbar^2}{4},$$

and therefore asked (2.145). □

At the time of the founding of quantum mechanics, in the first half of the 20th century, various proofs of the uncertainty principle were sought. The one in the example above is based on the mean of the indeterminacy squared:

$$\begin{cases} \overline{x^2} = \overline{(\Delta x)^2} = \int_{-\infty}^{\infty} x^2 |\psi|^2 \, dx \\ \overline{p^2} = \overline{(\Delta p)^2} = \hbar^2 \int_{-\infty}^{\infty} \left| \frac{d\psi}{dx} \right|^2 dx \end{cases} \tag{2.147}$$

and on the quadratic equation. The following is based on the *Hölder's inequality*

$$\int_X |f|^2\,dx \cdot \int_X |g|^2\,dx \geq \left|\int_X f^* g\,dx\right|^2, \tag{2.148}$$

in this form also called *Bunyakovsky*[34]*-Schwartz's*[35] *inequality.*

Let's start with two linear operators $\hat{A}$ and $\hat{B}$, from the expression:

$$\overline{(\Delta A)^2} = \int_X \psi^* |\hat{A} - \bar{A}|^2 \psi\,dx, \quad \overline{(\Delta B)^2} = \int_X \psi^* |\hat{B} - \bar{B}|^2 \psi\,dx, \tag{2.149}$$

so let's put it $\hat{A} - \bar{A} = \hat{a}$ and $\hat{B} - \bar{B} = \hat{b}$. We have:

$$\overline{(\Delta A)^2} = \int_X \psi^* |\hat{a}|^2 \psi\,dx = \int_X \hat{a}^* \psi^* \hat{a}\psi\,dx = \int_X |\hat{a}\psi|^2\,dx, \quad \overline{(\Delta B)^2} = \int_X |\hat{b}\psi|^2\,dx,$$

$$\overline{(\Delta A)^2} \cdot \overline{(\Delta B)^2} = \int_X |\hat{a}\psi|^2\,dx \cdot \int_X |\hat{b}\psi|^2\,dx,$$

$$\overline{(\Delta A)^2} \cdot \overline{(\Delta B)^2} \geq \left|\int_X \psi^* \hat{a}\hat{b}\psi\,dx\right|^2,$$

which follows from (2.148). How is it

$$\hat{a}\hat{b} = \frac{\hat{a}\hat{b} + \hat{b}\hat{a}}{2} + \frac{\hat{a}\hat{b} - \hat{b}\hat{a}}{2},$$

it is:

$$\overline{(\Delta A)^2} \cdot \overline{(\Delta B)^2} \geq \left|\int_X \psi^* \left(\frac{\hat{a}\hat{b} + \hat{b}\hat{a}}{2} + \frac{\hat{a}\hat{b} - \hat{b}\hat{a}}{2}\right)\psi\,dx\right|^2,$$

$$\overline{(\Delta A)^2} \cdot \overline{(\Delta B)^2} \geq \left|\int_X \psi^* \frac{\hat{a}\hat{b} + \hat{b}\hat{a}}{2}\psi\,dx + \int_X \psi^* \frac{\hat{a}\hat{b} - \hat{b}\hat{a}}{2}\psi\,dx\right|^2,$$

$$\overline{(\Delta A)^2} \cdot \overline{(\Delta B)^2} \geq \left|\int_X \psi^* \frac{\hat{a}\hat{b} - \hat{b}\hat{a}}{2}\psi\,dx\right|^2.$$

Operators under integrals are self-conjugated, which means that the mean values can be considered as even numbers, say $2m$ and $2n$ where $I \geq m^2 + n^2$ and $I \geq n^2$. This inequality becomes equality when the functions are proportional, when

$$\hat{b}\psi = C\hat{a}\psi,$$

and then the substitution gives

$$(C^* + C)\overline{\hat{a}^2} = 0,$$

so $C^* = -C$, because $\overline{\hat{a}^2}$ is not equal to zero. According, C is a purely imaginary constant and we can put $C = i\beta$ where β is real constant.

In addition, we calculate the commutator:

$$[\hat{a}, \hat{b}] = \hat{a}\hat{b} - \hat{b}\hat{a} = \cdots = \hat{A}\hat{B} - \hat{B}\hat{A} = [\hat{A}, \hat{B}].$$

[34]Viktor Bunyakovsky (1804-1889), Russian mathematician.
[35]Hermann Schwarz (1843-1921), German mathematician.

So, we have:

$$\overline{(\Delta A)^2} \cdot \overline{(\Delta B)^2} \geq \left| \int_X \psi^* \frac{\hat{A}\hat{B} - \hat{B}\hat{A}}{2} \psi \, dx \right|^2 = \left| \frac{\hat{A}\hat{B} - \hat{B}\hat{A}}{2} \right|^2 . \tag{2.150}$$

This is a very important result for understanding the principle of uncertainty!

The product of uncertainty of linear operators cannot be less than the absolute value of half of their commutator

$$\sqrt{\overline{(\Delta A)^2}} \cdot \sqrt{\overline{(\Delta B)^2}} \geq \left| \frac{[\hat{A}, \hat{B}]}{2} \right| . \tag{2.151}$$

This general term, which justifies the name *uncertainty principle*, is followed by Heisenberg's relations of uncertainty, then by the conviction that they come from noncommutative processes, and that they are also an expression of quantum entanglement. For the latter, we note that the over-involvement of the background laws of physics in the measurement process interferes with the extreme accuracy of the measurement itself. It is as if the materially where secondary to the abstract.

2.10.6 Momentum and Energy operators

It is clear from the notation of the Schrödinger equation that the *energy operator* is

$$\hat{E} = i\hbar \frac{\partial}{\partial t}, \tag{2.152}$$

where $\hbar$ is Planck's reduced constant, i is an imaginary unit, and partial derivatives (denoted by ∂) are used instead of total derivatives (d/dt) because the wave function $\psi(\mathbf{r}, t)$ is also a function of the position $\mathbf{r} = \mathbf{r}(x, y, z)$. *Momentum operator*, along the abscissa, ordinates and applicate, is:

$$\hat{p}_x = -i\hbar \frac{\partial}{\partial x}, \quad \hat{p}_y = -i\hbar \frac{\partial}{\partial y}, \quad \hat{p}_z = -i\hbar \frac{\partial}{\partial z}. \tag{2.153}$$

In analogy to classical mechanics, where the Hamiltonian is the sum of the kinetic and the potential energy of the system, here we have *operator Hamiltonian*

$$\hat{H} = \hat{T} + \hat{V}, \tag{2.154}$$

where

$$\hat{T} = \frac{\hat{p}^2}{2m} = -\frac{\hbar^2}{2m} \nabla^2 \tag{2.155}$$

operator *kinetic energy*, so $\hat{p} = -i\hbar\nabla$ is the momentum operator (all three coordinates), and $\hat{V} = V = V(\mathbf{r}, t)$ is the *potential energy* operator.

First example are position and momentum operators. Now we have:

$$\sqrt{\overline{(\Delta \hat{x})^2}} \cdot \sqrt{\overline{(\Delta \hat{p})^2}} \geq \left| \frac{[\hat{x}, \hat{p}]}{2} \right| = \left| \frac{i\hbar}{2} \right| = \frac{\hbar}{2},$$

and these are the known Heisenberg relations of uncertainty (2.145).

Second example are energy and time operators:

$$[\hat{E}, \hat{t}]\psi = (\hat{E}\hat{t} - \hat{t}\hat{E})\psi = i\hbar \frac{\partial}{\partial t}(t\psi) - ti\hbar \frac{\partial}{\partial t}\psi = i\hbar\psi,$$

hence the term for commutator

$$[\hat{E}, \hat{t}] = i\hbar. \tag{2.156}$$

Now from (2.151) we get

$$\sqrt{(\Delta\hat{E})^2} \cdot \sqrt{(\Delta\hat{t})^2} \geq \frac{\hbar}{2}, \tag{2.157}$$

i.e. the known uncertainty relations for energy and time.

2.10.7 Other examples

You can see a very interesting explanation of this relationship in the popular documentary "Everything and Nothing" by Professor Al-Khalili[36]. It takes two files of equal data, the first representing a picture of a pool table and the second is a movie on which the balls roll. The first is a sharp image of a large resolution of motionless balls that we do not see where they are going and with what energy, and the second is video of same moving balls but each image of a small resolution, blurry when we try to enlarge it.

The uncertainty (2.139) allows *vacuum* to be *dynamic void*. At sufficiently short time intervals Δt in vacuum, *virtual particles* of ΔE energy can be randomly generated, provided that $\Delta E \Delta t < \hbar/2$, for which all conservation laws apply. The appearance of these particles was experimentally confirmed by the rocking of the real particles as they passed through the vacuum.

Clearly, different events, A and B, may or may not have the same capabilities, same outcomes. For example: I was able to enroll in college yesterday, but it's too late today. Or, students Alice and Bob took the same exam but did not show the same knowledge. It is also clear that the possibilities of the same events change over time, with the circumstances, their actions taken.

The common possibilities of given events are their intersection, $A \cap B$, and the outcomes, say impossible for A, are in the set difference $B \backslash A$. The evolutions of quantum states, along the uncertainty relations, change these chances as well. It is strange, but no more of that, the statement that the substance moved from the past to the future becomes less informative. The uncertainty relations are not only about the non-commutative of change, where the đamagečaused by the first change will be irreparable to the second and vice versa, but also about reversing the flow of change. In the order A then B of an event there is an excess of action in relation to the order of B then A of the same. In this sense, the uncertainty relations are a direct confirmation of the principle of information.

Aside, I leave the question of resolving the *paradox of information* that, because of principle minimalism, the world becomes less informative over time. A similar conclusion follows from the spontaneous growth of entropy. This is then inconsistent with the law of information retention, unless we accept the hypothesis that space accumulates that difference. I wrote about this earlier and would not repeat it now.

2.11 Compton's rejection

The Compton Effect is another confirmation of Heisenberg's relations of uncertainty, which preceded them. But here we will not deal with this aspect of it but the discovery that light is of a particulate nature, since electron-deflected photons increase the wavelength according to the laws that could apply to billiard balls in a collision. Recalls, *reflection*

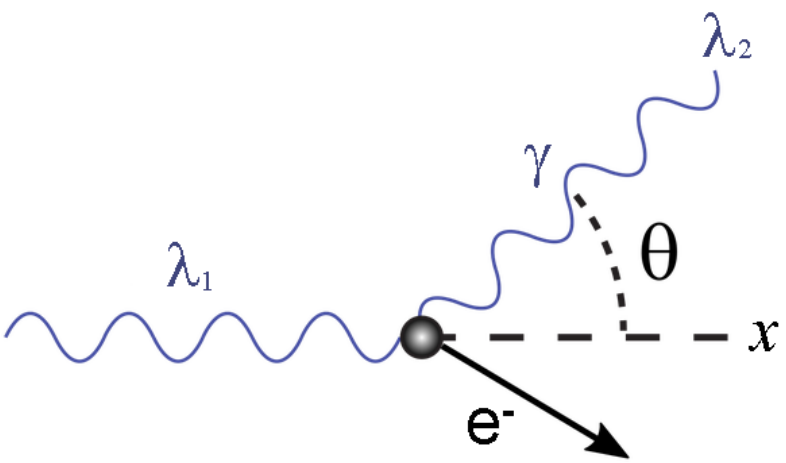
[36] Jim Al-Khalili, b. 1962 in Iraq, British physicist.

involves a change in direction of waves when they bounce off a barrier, when the angle of incidence equals the reflection angle and the wavelength remains the same. *Refraction* of waves involves a change in the direction of waves as they pass from one medium to another. Refraction, or the bending of the path of the waves, is accompanied by a change in speed and wavelength of the waves.

Sometime in the early 1920s, when light and photon waves were still discussed based on photoelectric effects, Compton gave clear experimental evidence of the "particle nature" of these waves. He observed the scattering of γ-rays from carbon electrons and found scattered γ-rays with larger wavelengths than incident ones, as in the picture 2.11. The change in wavelength increases with the scattering angle according to the Compton formula

$$\lambda_2 - \lambda_1 = \Delta\lambda = \frac{h}{m_e c}(1 - \cos\theta). \tag{2.158}$$

On the left side of the equality are λ_1 input and λ_2 output wavelength of scattered photons. To the right is approximately $h = 6.626 \times 10^{-34}$ Js Planck's constant, $m_e = 9.109 \times 10^{-31}$ kg is the *electron mass* at rest, $c = 3 \times 10^8$ m/s *speed of light* in vacuum. The *scattering angle* is θ, and the quotient $h/m_e c = 2.43 \times 10^{-12}$ m is called the Compton electron wavelength.



Slika 2.11: Compton effect.

Compton processed and explained the data from his experiment, applying the laws of energy and momentum conservation in the interactions of photons γ and electrons e^-, and obtained that the scattered photon had less energy and greater wavelength according to Planck[37] formulas

$$E = h\nu, \tag{2.159}$$

where $\nu = c/\lambda$ is the frequency of the wave. Here's the calculation.

The photon γ with initial energy $E_1 = h\nu_1$ travels along the abscissa (x-axis) and scatters on the electron e^- of energy $E_e = m_e c^2$, as in the picture 2.11. The photon bounces off the electron in a direction that closes the angle θ with the abscissa, leaving with energy $E_2 = h\nu_2$. We know that the photons wavelength λ and the frequency ν are related by $\lambda\nu = c$. The electron energy at rest is $E_{e_1} = m_e c^2$, and after collision $E_{e_2} = \sqrt{p_e^2 c^2 + m_e^2 c^4}$, where $\vec{p}_e$ is the electron momentum after the collision.

[37]Max Planck (1858-1947), German theoretical physicist.

We apply the *conservation law* for energy and calculate:

$$E_1 + E_{e1} = E_2 + E_{e2},$$

$$E_1 + m_e c^2 = E_2 + \sqrt{p_e^2 c^2 + m_e^2 c^4},$$

$$(E_1 - E_2 + m_e c^2)^2 = p_e^2 c^2 + m_e^2 c^4.$$

Applying the conservation law for momentum, we obtain:

$$\vec{p}_1 = \vec{p}_2 + \vec{p}_e,$$

$$(\vec{p}_1 - \vec{p}_2)^2 = \vec{p}_e^2,$$

$$p_1^2 + p_2^2 - 2p_1 p_2 \cos\theta = p_e^2.$$

Multiplying this equality by c^2 and using the relation $E = pc$ for a photon, it becomes

$$E_1^2 + E_2^2 - 2E_1 E_2 \cos\theta = p_e^2 c^2.$$

Combining this result with the previous one, we find:

$$E_1^2 + E_2^2 + m_e^2 c^4 + 2E_1 m_e c^2 - 2E_2 m_e c^2 - 2E_1 E_2 = E_1^2 + E_2^2 - 2E_1 E_2 \cos\theta + m_e^2 c^4,$$

$$E_1 m_e c^2 - E_2 m_e c^2 - E_1 E_2 = -E_1 E_2 \cos\theta,$$

$$\frac{1}{E_2} - \frac{1}{E_1} = \frac{1}{m_e c^2}(1 - \cos\theta),$$

$$\lambda_2 - \lambda_1 = \frac{h}{m_e c}(1 - \cos\theta),$$

which is the formula (2.158). From the previous one we also get the formula for the scattered photon energy

$$E_2 = \frac{E_1}{1 + \frac{E_1}{m_e c^2}(1 - \cos\theta)}, \tag{2.160}$$

where E_1 is the energy of the incident photon. How is $|\cos\theta| \leq 1$ this denominator to the right is not less than one, so $E_2 \leq E_1$, and equality is reached only when there is no scatter, $\theta = 0$.

Compton[38] for his discovery in 1927 was awarded the Nobel Prize in Physics and is now a classic. What is less well known is the relationship between wavelength, amplitude, and probability, which I look at (hypothetically) here in the previously mentioned and used ways[39]. In principle, I believe that a photon travels its own trajectory that seems most likely to it, and that by colliding with an electron, the photon diverts to a less likely path. In addition, I consider the wavelength to be an indicator of the probability of the trajectory, in a way that larger wavelengths correspond to lower photon position densities.

[38] Arthur Compton (1892-1962), American physicist.
[39] see in previous books like [3]

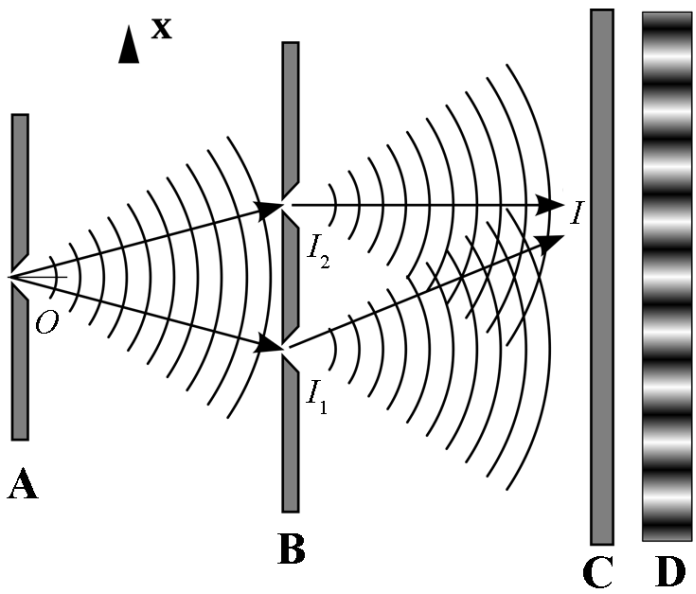
2.12 Double-slit experiment

Figure 2.12 shows the experiment double-slit. The waves (light) start from position O on the wall A, pass through two narrow openings I_1 and I_2 at obstacle B and reach the surface C that registers them, forming a time diffraction pattern D. We look at one place I at curtain C and at that point the interference of two waves I_1 and I_2.

When one of the two openings is closed there is no *interference*. The experiment shows that waves come from O, passed through that single opening I_i at obstacle B and cluster at curtain C, most intensely at penetrating true OI through plane C, gradually reducing the intensity of accumulation at further places in that plane. The classic explanation would be that there is no interference because it requires at least two waves, and more recently these are *probability waves*. It is informatics explanation that the particle-wave ray, passing through only one opening, declares itself, loses some of its uncertainty and becomes less "wave" and more "particle".

If both openings I_1 and I_2 are free, the waves OI_1 and OI_2 after passing through B interfere and this is seen in the form of diffraction bands D on the detection plate C. This is also noted in the experiment and explains how according to classical physics, probabilities, and (my) information theory.

Things get weird for classical physics when we miss one by one particle-wave from O in the direction of two openings of obstacle B. That is when diffraction lines D also appears on screen C as if the interference of the present particle-wave with the previous ones, now nonexistent is happening.



Slika 2.12: Experiment "double-slit".

Classical physics cannot explain such a ghost interference. Recent quantum mechanics explains it by scattering the "probability wave" of each particle through two openings and interfering those parts of one particle with itself. Quantification is debatable then, so I offer another, IT explanation. Space is the storehouse of the past; it is the condenser of the present. Each place remembers the waves that have passed it and in the present there are corresponding interferences.

In other words, the waves we see in the given place now are the interferences of all the corresponding waves that have passed there in the past, such as the waves on the ocean surface that result and the large deposits of water below. This attitude can also be justified by timeless wave functions.

Denote by $A = A(I)$ the outcome amplitude and by $A_1 = A(I_1)$ and $A_2 = A(I_2)$ the amplitudes of the waves passing through the openings. Quantum probability is, according to Born rule, the square of the amplitude $P(I) = |A|^2 = A^*A$, where A^* is a conjugate complex number of A. Denote by $\Re(A), \Im(A) \in \mathbb{R}$, respectively, the real and imaginary part of the complex number $A = \Re(A) + i\Im(A)$ ($i^2 = -1$) so let's check:

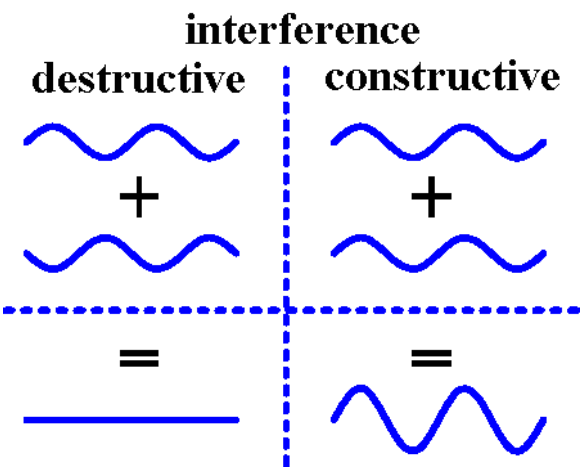
$$\begin{cases} P(I) = P(I_1 \cup I_2) = P(I_1) + P(I_2) - P(I_1 \cap I_2) \\ |A|^2 = |A_1 + A_2|^2 = (A_1 + A_2)^*(A_1 + A_2) = A_1^*A_1 + A_1^*A_2 + A_2^*A_1 + A_2^*A_2. \end{cases} \tag{2.161}$$

The expression below, $|A|^2 = |A_1|^2 + |A_2|^2 + 2\Re(A_1^*A_2)$, after comparing with the above gives:

$$P(I) = |A|^2, \quad P(I_1) = |A_1|^2, \quad P(I_2) = |A_2|^2, \quad P(I_1 \cap I_2) = 2\Re(A_1^*A_2), \tag{2.162}$$

where $\Re(A_1^*A_2) = \Re(A_1)\Re(A_2) + \Im(A_1)\Im(A_2)$. As the amplitudes of the waves arriving through the openings change depending on the position along the surface C, so does the real part of the product, and with it the total probability of energy coming to a given place of the measuring surface in the previous figure. This is an experimentally proven result.

In the following picture 2.13 we see what happens when the wave amplitudes are opposite and when they are in agreement. In the first case (left), the real and imaginary parts of the amplitudes have the opposite sign, which is why $\Re(A_1^*A_2) < 0$ and the probabilities of occurrence on the shutter C decrease. In the second case (right), the real and imaginary parts of the amplitudes have the same sign, $P(A_1^*A_2) > 0$, so the probabilities at shutter C increase. Hence the dark and light diffraction stripes (D). It is strange here, when compared to the expression above (2.162), that the probability of associated events may be greater than the sum of the individual ones.



Slika 2.13: Constructive and destructive interference.

The destructive and constructive phase of interference with D-diffraction bands would not be a novelty for classical physics if it were not *synergy*, the sum of associates larger than the sum of the items themselves. This is unusual in the particle-wave interference formulas

(2.161) for the double-opening experiment. This synergy further indicates that something else "from below" is coming up to the "surface".

2.13 Quantum entanglement

As with the previous cover topics, I have written more about *quantum entanglement* earlier, so here I will just point out a few places to complement the popular, the first part of this book.

Quantum "conjunction" occurs when a system of multiple particles in quantum mechanics acts in such a way that the particles cannot be described as independent systems but only as one system as a whole. We are talking about random events and probabilities. Dependent events are, for example, the storm and departure of airplanes from the airport, or the withdrawal of tickets from the pack without returning. The individual outcomes of probability distribution are independent, but each is dependent on the sum of all others.

Example 2.13.1. *From a standard 52-card deck, one card is randomly drawn (event A) and then another one (event B) without returned the previous. What is the probability that the first card is a "queen" and the second is a "jack"?*

Solution. $\Pr(A) = \frac{4}{52}$, $\Pr(B) = \frac{4}{51}$, $\Pr(A, B) = \frac{4 \cdot 4}{52 \cdot 51} = \frac{4}{663}$. □

Unlike event stringing, the *event product* (intersecting sets) is an event that will happen iff[40] each occurs. For example, if A is an event that the card "ace" is drawn, and B is an event that the "spades" is drawn, then AB is the event that the "ace spade" is drawn. With $n(X)$ denote the relative frequency of events X in $n \in \mathbb{N}$ repetitions of the random experiment.

For number $\Pr(B|A)$, that we call *conditional probability* of the event B with the conditions A, if before B happend A, then is valid the equations:

$$\Pr(B|A) = \frac{\Pr(AB)}{\Pr(A)} = \frac{n(AB)}{n(A)} = \frac{\frac{n(AB)}{n}}{\frac{n(A)}{n}}, \tag{2.163}$$

if $\Pr(A) > 0$. From the commutativity $AB = BA$ follows

$$\Pr(AB) = \Pr(A)\Pr(B|A) = \Pr(B)\Pr(A|B). \tag{2.164}$$

Disjoint events A and B are those whose product is an empty set ($AB = \varnothing$).

We believe that there are non-empty events $X \neq \varnothing$ that (under the given circumstances) are impossible $\Pr(X) = 0$, that from $\Pr(\varnothing) = 0$ does not necessarily follow $X = \varnothing$. Event complement of A is event A' that will happen iff A will not happen. Complementing the impossible gives the certain event ($\varnothing' = \Omega$); impossible event has probability 0, certain 1.

Event sum (union of events) is an event that happens if at least one of them happens. If A is an event that a card "ace" was drawn, and B is an event that a "spade" was drawn, then $A + B$ is the event that the drown card was any of the four "aces" or any of the 13 "spades", which is a total of 16 out of 52.

Breaking a certain event Ω into a series of disjoint events $A_1, A_2, \ldots, A_n$ we get the formula of *total probability*

$$\Pr(B) = \sum_{k=1}^{n} \Pr(A_k)\Pr(B|A_k). \tag{2.165}$$

[40] iff – if and only if

Specifically, the result follows from $B = \sum_{k=1}^{n} A_k B$, whence $\Pr(B) = \sum_{k=1}^{n} \Pr(A_k B)$, then putting $\Pr(A_k B) = \Pr(A_k)\Pr(B|A_k)$.

Hence the *Bayes' theorem*: if events $A_1, A_2, \ldots, A_n$ make one break of Ω

$$\Pr(A_l|B) = \frac{\Pr(A_l)\Pr(B|A_l)}{\sum_{k=1}^{n}\Pr(A_k)\Pr(B|A_k)}, \quad l = 1, 2, \ldots, n. \tag{2.166}$$

Namely, from $\Pr(A_l B) = \Pr(B)\Pr(A_l|B) = \Pr(A_l)\Pr(B|A_l)$ follows

$$\Pr(A_l|B) = \frac{\Pr(A_l)\Pr(B|A_l)}{\Pr(B)}$$

and supstituting $\Pr(B)$ by formula of total probability we get the required result.

Event B is *independent* of event A in terms of probability theory if

$$\Pr(B|A) = \Pr(B). \tag{2.167}$$

Independent events can be defined also with $\Pr(AB) = \Pr(A)\Pr(B)$.

Example 2.13.2. *One of 32 is accidentally drawn from a deck of 32 cards. Are events: A drawn "spade" and event B drawn "queen" independent?*

Solution. It's not obvious[41], so we're calculating:

$$\Pr(A) = \frac{8}{32} = \frac{1}{4}, \quad \Pr(B) = \frac{4}{32} = \frac{1}{8}, \quad \Pr(AB) = \frac{1}{32},$$

$$\Pr(A)\cdot\Pr(B) = \Pr(AB).$$

So A and B events are independent in terms of the definition above. □

Example 2.13.3. *A regular tetrahedron has one side painted blue, the other yellow, the third red and the fourth with all three colors. The tetrahedron is thrown and the color on the side to which it has fallen is registered. Let the event be: A to drop blue, B yellow, C red. Check independence.*

Solution. We find easy:

$$\Pr(A) = \Pr(B) = \Pr(C) = \frac{2}{4} = \frac{1}{2}, \quad \Pr(AB) = \Pr(BC) = \Pr(CA) = \frac{1}{4},$$

$$\Pr(A)\Pr(B) = \Pr(AB), \quad \Pr(B)\Pr(C) = \Pr(BC), \quad \Pr(C)\Pr(A) = \Pr(CA),$$

$$\Pr(A)\Pr(B)\Pr(C) \neq \Pr(ABC).$$

The three events A, B, and C are not collectively independent, although they are independent in pairs. □

This example shows that three or more events independent in pairs need not be independent in terms of mathematical definition. It is similar to the aforementioned probability distribution whose individual events are independent in pairs but not in terms of the upper definition. This is an important observation, although independence is defined in an

[41] examples from [19].

"obvious" and intuitively acceptable way (2.167), it quickly leads us to intuitively hard to understand results.

The difficulties with quantum entanglement come from dependent random events. The book Physical Information [2] demonstrates how Shannon's definition of information shorten the assessment of physical information, precisely because of that excess of certainty, that is, the deficit of information that arises from the latent dependence of probability distribution events, which is why in appropriate quantum phenomena happened "excess action".

Einstein's famous example with a *pair of gloves*, "left" and "right", in two very far boxes. If we open closer to us and find say "left", at the moment we understand that in the far box is "right". That are dependent events and "surplus action", which is quoted because there is no action. Any information is an action and vice versa, so there is no action unless it has information.

If we know something for certain, the probability of the event is 1 and information is 0. Opening the first box consumes all the uncertainty from the events, obtains all possible information and performs all possible action, and the possible problem is only in the difficulty of our intuition to accept this *non-locality*, more precisely said with the dependence of random events.

A special type of dependent events, and therefore entangled quantum systems, is *perception information* in the case of a physical substance. Then the very demanding principle of least action applies, which means that acting on one of the vectors (intelligence or hierarchy) causes the other to be tuned so that their scalar product remains minimal.

2.14 Anticommutativity

One *antisymmetric* two-particle state $\hat{A}(x,y)$ in quantum mechanics is represented as the sum of states in which one particle is in the state $|x\rangle$ and the other in state $|y\rangle$, so

$$|\psi\rangle = \sum_{x,y} \hat{A}(x,y)|x,y\rangle, \tag{2.168}$$

and $\hat{A}(y,x) = -\hat{A}(x,y)$. Hence, if $x = y$, then $\hat{A}(x,y) = 0$, which is Pauli's exclusion[42].

This antisymmetry is also called *anticommutativity*. Consider it on the helium atom in the image 2.14. Electrons 1 and 2 are in states a and b respectively, and the wave function for individual electrons is $\psi_1(a)$ and $\psi_2(b)$.

The probability (density) of the amplitude that an electron 1 or 2 is in the state a or b is respectively $\|\psi_1^*(a)\psi_1(a)\|$ or $\|\psi_2^*(b)\psi_2(b)\|$. But the amplitude probability that electron 1 is in state a and (conjunction) electron 2 is in state b is

$$\psi = \psi_1(a)\psi_2(b). \tag{2.169}$$

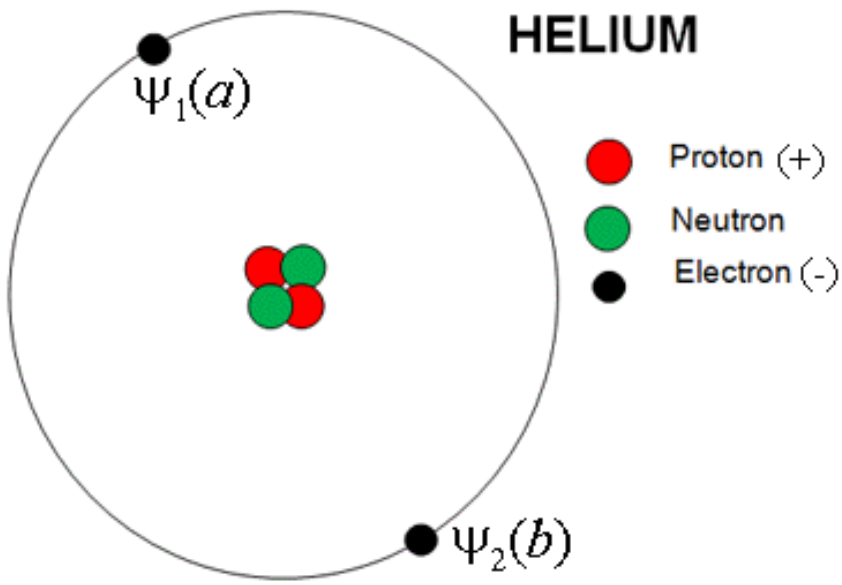
We can write the wave function state with both states a and b occupied by electrons

$$\psi(a,b) = \psi_1(a)\psi_2(b) - \psi_1(b)\psi_2(a). \tag{2.170}$$

We see that $\psi(b,a) = -\psi(a,b)$, which is anti-symmetry or anti-commutativity.

Since electrons are fermions, the sign between (2.170) is minus. In the case of bosons instead of minus there would be a plus and we would have symmetry or commutativity. In the case of fermions we have the exclusion of Pauli, in the case of bosons no.

[42] see [15]



Slika 2.14: The helium atom has two electrons.

Comparing this with the information of perception, we assume that its items (2.169) are some information and that such equivalents are the logarithm of probability, that is, the probability is the exponent of the information. Thus we see state (2.169) as the product of the exponents of the information of electron 1 in the state a and electron 2 in the state b. Due to the anti-commutative (2.170) information of electrons is smaller, and because of the principle of minimalism they remain trapped in this state.

2.14.1 Pauli matrices

Pauli matrices are still the most famous antisymmetric quantum processes, so I specifically mention them:

$$\hat{\sigma_x} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \ \hat{\sigma_y} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \ \hat{\sigma_z} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.171}$$

They have already been mentioned here (2.131) in a slightly different context, and now we pay attention to the anti-symmetry of the first two and the trace (sum of diagonal coefficients) of the third:

$$\hat{\sigma_x}\hat{\sigma_y} = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix} = -\hat{\sigma_y}\hat{\sigma_x}, \quad \mathrm{Tr}[\hat{\sigma_z}] = 0.$$

That is why any two of the three are anti-commutative and may represent elemental fermions.

These three matrices are *involutory*, $f(f(x)) = x$, or

$$\hat{\sigma}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \hat{I}.$$

The eigenvalues of each of the Pauli matrices are ±1. The corresponding normalized eigenvectors are:

$$\hat{\psi}_{x+} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\1\end{pmatrix}, \quad \vec{\psi}_{x-} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-1\end{pmatrix},$$
$$\hat{\psi}_{y+} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\i\end{pmatrix}, \quad \vec{\psi}_{y-} = \frac{1}{\sqrt{2}}\begin{pmatrix}1\\-i\end{pmatrix},$$
$$\hat{\psi}_{z+} = \begin{pmatrix}1\\0\end{pmatrix}, \quad \vec{\psi}_{z-} = \begin{pmatrix}0\\1\end{pmatrix}.$$

Note that each of these three vectors is an *unit vector*. For example, $\|\vec{\psi}_{x+}\|^2 = \frac{1}{2}(1+1) = 1$ applies to the first.

We also have Pauli *rotated matrices*, defined by any three unit vectors $\vec{n}$ in the direction (θ, ϕ) of the polar system $O\theta\phi$. Written covariantly, for unit vector $\vec{u} = (u_x, u_y, u_z)$ due to $u_x^2 + u_y^2 + u_z^2 = 1$ is valid

$$\vec{u} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta). \tag{2.172}$$

An inner (scalar) product with a counter-variant matrix vector $\vec{\sigma} = (\hat{\sigma}_x, \hat{\sigma}_y, \hat{\sigma}_z)^\tau$ is

$$\vec{u}\cdot\vec{\sigma} = \hat{\sigma}_x \sin\theta\cos\phi + \hat{\sigma}_y \sin\theta\cos\phi + \hat{\sigma}_z\cos\theta.$$

Subtracting matrices, we get after addition

$$\vec{u}\cdot\vec{\sigma} = \begin{pmatrix}\cos\theta & e^{-i\phi}\sin\theta\\ e^{i\phi}\sin\theta & -\cos\theta\end{pmatrix}. \tag{2.173}$$

Let's notice

$$\begin{aligned} e^{\pm i\phi}\sin\theta &= (\cos\phi \pm i\sin\phi)\sin\theta \\ &= \cos\phi\sin\theta \pm i\sin\phi\sin\theta \\ &= u_x \pm iu_y. \end{aligned}$$

Hence

$$\vec{u}\cdot\vec{\sigma} = \begin{pmatrix}n_z & n_x - in_y\\ n_x + in_y & -n_z\end{pmatrix}. \tag{2.174}$$

The spin must be the eigenstate of the rotated Pauli matrix $(\vec{n}\cdot\vec{\sigma})$ with the unit eigenvalue:

$$\begin{pmatrix}n_z & n_x - in_y\\ n_x + in_y & -n_z\end{pmatrix}\begin{pmatrix}\xi_1\\ \xi_2\end{pmatrix} = \begin{pmatrix}\xi_1\\ \xi_2\end{pmatrix}.$$

By multiplying and editing the system, we get:

$$\begin{cases} n_z\xi_1 + n_x\xi_2 - in_y\xi_2 = \xi_1 \\ n_x\xi_1 + in_y\xi_1 - n_z\xi_2 = \xi_2, \end{cases}$$

$$\begin{cases} (n_z - 1)\xi_1 + (n_x - in_y)\xi_2 = 0 \\ (n_x + in_y)\xi_1 - (n_z + 1)\xi_2 = 0, \end{cases}$$

$$\begin{cases} (\cos\theta - 1)\xi_1 + (e^{-i\phi}\sin\theta)\xi_2 = 0 \\ (e^{+i\phi}\sin\theta)\xi_1 - (\cos\theta + 1)\xi_2 = 0, \end{cases}$$

$$\begin{cases} (-2\sin^2\frac{\theta}{2})\xi_1 + (e^{-i\phi}2\sin\frac{\theta}{2}\cos\frac{\theta}{2})\xi_2 = 0 \\ (e^{+i\phi}2\sin\frac{\theta}{2}\cos\frac{\theta}{2})\xi_1 - (2\cos^2\frac{\theta}{2})\xi_2 = 0, \end{cases}$$

$$\begin{cases} (-\sin\frac{\theta}{2})\xi_1 + (e^{-i\phi}\cos\frac{\theta}{2})\xi_2 = 0 \\ (e^{i\phi}\sin\frac{\theta}{2})\xi_1 - (\cos\frac{\theta}{2})\xi_2 = 0. \end{cases}$$

This is a dependent consistent linear system with two equations and two complex variables ξ_1 and ξ_2. It has infinite solutions.

Let's take an arbitrary complex number $\xi_1 \in \mathbb{C}$ and compute $\xi_2 = e^{i\phi}\tan\frac{\theta}{2}\xi_1$ with the condition $\|\xi_1\|^2 + \|\xi_2\|^2 = 1$. The general solution is

$$\vec{\psi} = \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} = \begin{pmatrix} e^{-i\phi/2}\cos(\theta/2) \\ e^{i\phi/2}\sin(\theta/2) \end{pmatrix}. \tag{2.175}$$

It is an eigenvector of a rotated Pauli matrix (2.174) to which eigenvalue 1 corresponds.

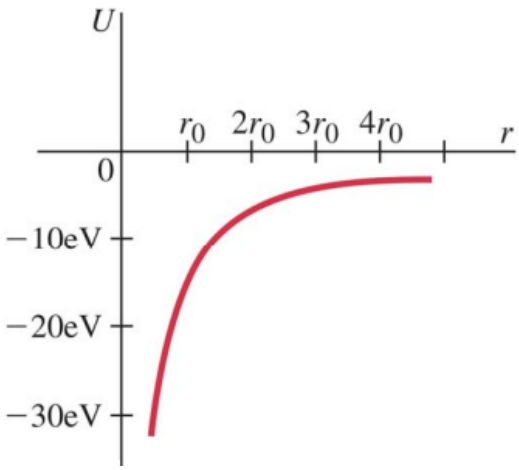
Note that the 2π rotation gives $\vec{\psi} \to -\vec{\psi}$. To return the spin to its initial value, two full rotations of the vector (2.174) are required. Only after double full rotation, for the angle 4π, does the spin become the same as at the beginning. We simulate this with examples such as Dirac's belt[43], and its deeper meaning lies in the ability of fermions to have *antiparticles*. Bosons, on the contrary, have no antiparticles, so for example, a photon is said to be both a particle and an antiparticle.

2.15 Solving Hydrogen Atom

The simplest physical system that contains potential interactions (not an isolated particle) is the *hydrogen atom*, which we will now consider as solutions to the Schrödinger equation (2.100). One proton, one electron, and an electrostatic *Coulomb's potential* holding them together

$$U = -\frac{e^2}{4\pi\varepsilon_0 r}, \tag{2.176}$$

which is the attractive potential between the charge $+e$ and $-e$ at the distance r.



Slika 2.15: Coulomb's potential.

[43]Dirac's belt: `https://vimeo.com/62228139`

Transforming the coordinates between Descartes $Oxyz$ and the spherical $Or\theta\phi$ system yields *Schrödinger equation* in Descartes

$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}+\frac{2m}{\hbar^2}(E-U)\psi=0, \tag{2.177}$$

and spherical coordinates

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{1}{r^2\sin^2\theta}\frac{\partial^2\psi}{\partial\phi^2}+\frac{2m}{\hbar^2}(E-U)\psi=0. \tag{2.178}$$

When we include Coulomb's potential and multiply both sides of this equality by $r^2\sin^2\theta$, we get

$$\sin^2\theta\frac{\partial}{\partial r}\left(r^2\frac{\partial\psi}{\partial r}\right)+\sin\theta\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\psi}{\partial\theta}\right)+\frac{\partial^2\psi}{\partial\phi^2}+\frac{2mr^2\sin^2\theta}{\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0 r}+E\right)\psi=0. \tag{2.179}$$

The solution to this equation is the wave function ψ for an electron in a hydrogen atom. If we find that solution, in principle, we will find out everything we need about the hydrogen atom.

Equations like the above are solved, among other ways, by *separating variables*, i.e. by dividing it into different parts with one variable in each part. To this end, we write

$$\psi=\psi(r,\theta,\phi)=R(r)\Theta(\theta)\Phi(\phi)=R\Theta\Phi. \tag{2.180}$$

Then we look at the derivate:

$$\frac{\partial\psi}{\partial r}=\Theta\Phi\frac{dR}{dr},\quad \frac{\partial\psi}{\partial\theta}=R\Phi\frac{d\Theta}{d\theta},\quad \frac{\partial^2\psi}{\partial\phi^2}=R\Theta\frac{d^2\Phi}{d\phi^2}, \tag{2.181}$$

where partial statements have become full statements, because the functions R,Θ,Φ depend only on r,θ,ϕ respectively. We substitute this to the previous one.

Substitution $\psi=R\Theta\Phi$ into the Schrödinger equation and divide by $R\Theta\Phi$ gives

$$\frac{\sin^2\theta}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{\sin\theta}{\Theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)+\frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2}+\frac{2mr^2\sin^2\theta}{\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0 r}+E\right)=0. \tag{2.182}$$

The variable ϕ is separated so that the expression

$$\frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2}$$

is function only of the ϕ. Moving that to the right side of the equation, we get

$$\frac{\sin^2\theta}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{\sin\theta}{\Theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)+\frac{2mr^2\sin^2\theta}{\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0 r}+E\right)=-\frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2}. \tag{2.183}$$

The equation becomes of the form $f(r,\theta)=g(\phi)$, where f is a function of only r and θ, and g is a function of only the variable ϕ. This is possible only in the case of $f(r,\theta)=\text{const}$, hence the function f is a constant, that is, it is independent of r and θ. That's why we write

$$-\frac{1}{\Phi}\frac{d^2\Phi}{d\phi^2}=m_l^2, \tag{2.184}$$

and we'll see the meaning of m_l constant soon.

Divide equation (2.183) by $\sin^2\theta$, we obtain

$$\frac{1}{R}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\frac{2mr^2}{\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0 r}+E\right)=\frac{m_l^2}{\sin^2\theta}-\frac{1}{\Theta\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right), \tag{2.185}$$

so again we have split variables. On the left side of the equality is a function of only r and on the right is a function of θ itself. This is possible only if there are constants on each side of the equality, and in this case we will denote this constant by $l(l+1)$.

So we came up with three differential equations:

$$\begin{cases} \frac{d^2\Phi}{d\phi^2}+m_l^2\Phi=0 \\ \frac{1}{\sin\theta}\frac{d}{d\theta}\left(\sin\theta\frac{d\Theta}{d\theta}\right)+\left[l(l+1)-\frac{m_l^2}{\sin^2\theta}\right]\Theta=0 \\ \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right)+\left[\frac{2m}{\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0 r}+E\right)-\frac{l(l+1)}{r^2}\right]R=0. \end{cases} \tag{2.186}$$

Instead of one large partial differential equation (2.182) with three variables, we have three simpler differential equations with one variable each.

Quantum numbers are constants that define the solutions of the Schrödinger equation. In the case of the first equation (2.186) we have

$$\Phi(\phi)=Ae^{im_l\phi}, \tag{2.187}$$

where A is a norm constant. As ϕ and $\phi+2\pi$ represent one point in space, there will be

$$Ae^{im_l\phi}=Ae^{im_l(\phi+2\pi)},$$

which is true only for $m_l=0,\pm1,\pm2,\ldots$, and for reasons not yet visible at this point, we call m_l the *magnetic quantum number*.

The second equation (2.186) is by θ. It includes a member

$$l(l+1)-\frac{m_l^2}{\sin^2\theta}, \tag{2.188}$$

and it is shown that it can only be solved when l is an integer greater than or equal to m_l. Thus we find *orbital quantum number*, the l, with restriction $m_l=0,\pm1,\pm2,\ldots,\pm l$.

The third equation (2.186) is radial, its solutions are functions of the variable r. It can be solved only for energies E that satisfy some of the conditions we know from *Bohr*[44] Model of the Atom

$$E_n=-\frac{me^4}{32\pi^2n^2\varepsilon_0\hbar^2}=\frac{E_1}{n^2},\quad n=1,2,3,\ldots \tag{2.189}$$

This n is called *principal quantum number*. The product $l(l+1)$ appears in this third equation. It follows from the theory of differential equations that the solution R can be found only if n is greater than or equal to the number $l+1$, in other words, when the condition $l=0,1,2,\ldots,n-1$. This completes the states of the hydrogen atom.

Thus, the solutions of the Schrödinger equation (2.179) for a hydrogen atom must be of the form (2.180), with the specified properties of quantum numbers n, l, and m_l. The first few normalized solutions of these, the wave functions of the hydrogen atom, are given in table 2.1. The constant $a_0=4\pi\varepsilon_0\hbar^2/mc^2=5.292\times10^{-11}$ m is the radius of the inner Bohr

[44] Niels Bohr (1885-1962), Danish physicist.

n	l	m_l	$\psi(r,\theta,\phi)$
1	0	0	$\frac{1}{\sqrt{\pi}a_0^{3/2}}e^{-r/a_0}$
2	0	0	$\frac{1}{4\sqrt{2\pi}a_0^{3/2}}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$
2	1	0	$\frac{1}{4\sqrt{2\pi}a_0^{3/2}}\frac{r}{a_0}e^{-r/2a_0}\cos\theta$
2	1	±1	$\frac{1}{8\sqrt{\pi}a_0^{3/2}}\frac{r}{a_0}e^{-r/2a_0}\sin\theta\, e^{\pm i\phi}$
3	0	0	$\frac{1}{81\sqrt{3\pi}a_0^{3/2}}\left(27-18\frac{r}{a_0}+2\frac{r^2}{a_0^2}\right)e^{-r/3a_0}$
3	1	0	$\frac{\sqrt{2}}{81\sqrt{\pi}a_0^{3/2}}\left(6-\frac{r}{a_0}\right)\frac{r}{a_0}e^{-r/3a_0}\cos\theta$
3	1	±1	$\frac{1}{81\sqrt{\pi}a_0^{3/2}}\left(6-\frac{r}{a_0}\right)\frac{r}{a_0}e^{-r/3a_0}\sin\theta\, e^{\pm i\phi}$
3	2	0	$\frac{1}{81\sqrt{6\pi}a_0^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}(3\cos^2\theta-1)$
3	2	±1	$\frac{1}{81\sqrt{\pi}a_0^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}\sin\theta\cos\theta\, e^{\pm i\phi}$
3	2	±2	$\frac{1}{162\sqrt{\pi}a_0^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}\sin^2\theta\, e^{\pm 2i\phi}$

Tabela 2.1: Hydrogen atom states.

orbit. For example, for a quantum number $n = 3$ corresponding to the second excited state of a Bohr hydrogen atom and the orbital and magnetic numbers $l = 2$ and $m_l = -1$, we have normed solutions:

$$\Phi(\phi) = \frac{1}{\sqrt{2\pi}}e^{-i\phi}, \quad \Theta(\theta) = \frac{\sqrt{15}}{2}\sin\theta\cos\theta, \quad R(r) = \frac{4}{81\sqrt{30}a_0^{3/2}}\frac{r^2}{a_0^2}e^{-r/3a_0}, \tag{2.190}$$

and ψ is the product of these three. This is the penultimate row of the table.

The principal quantum number n depends on the energy of the electrons and is shown to be equal to the one previously known from the Bohr atom model. This equality comes from the wave nature of electrons, but with the additional details that the quantum solution brings us. It was known that electron energies in a hydrogen atom were quantified and had negative numbers, but now we know that positive energies can also be solutions to the Schrödinger equation, then for free electrons.

The differential equation for $R(r)$ can be written

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dR}{dr}\right) + \frac{2m}{\hbar^2}\left[K_r + K_o - \frac{\hbar^2 l(l+1)}{2mr^2}\right]R = 0, \tag{2.191}$$

where K_r and K_o are kinetic radial and orbital energies, with total kinetic energy $K = K_r + K_o$ and total energy $E = K + U$, with Coulomb potential energy (2.176). This kinetic energy is always positive. It is convenient to separate kinetic energies at K_r and K_o, because if the radial equation has only a radial dependence, then the member with orbital kinetic energy is negligible.

The ejection of orbital kinetic energy from Eq

$$K_o = \frac{\hbar^2 l(l+1)}{2mr^2}. \tag{2.192}$$

We know that $K_o = \frac{1}{2}mv_o^2$, where v_o is the orbital velocity (tangent), and that the orbital angular momentum is $L = mv_o r$, and hence $K_o = L^2/(2mr^2)$. Combining the previous ones, we find $L = \sqrt{l(l+1)}\,\hbar$, so since the number $l = 0, 1, 2, \ldots, n-1$, so it is quantized, so is L quantized. The lowest possible not null value is $L = \sqrt{2}\,\hbar$. The labels for l are in the table:

$$l = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ \text{s} & \text{p} & \text{d} & \text{f} & \text{g} & \text{h} & \text{i} \end{pmatrix}. \tag{2.193}$$

Thus, an electron with numbers $n = 3$, $l = 2$ would be a 3d electron. It is 3d with any of the few allowed numbers m_l. Some other states are in the table 2.2.

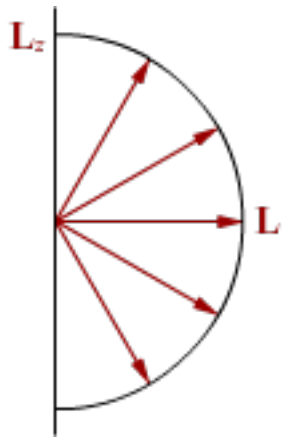
	$l = 0$	$l = 1$	$l = 2$	$l = 3$	$l = 4$	$l = 5$
$n = 1$	1s					
$n = 2$	2s	2p				
$n = 3$	3s	3p	3d			
$n = 4$	4s	4p	4d	4f		
$n = 5$	5s	5p	5d	5f	5g	
$n = 6$	6s	6p	6d	6f	6g	6h

Tabela 2.2: States of electrons in atom.

An electron in a hydrogen atom is in orbit. It has its angular momentum $\mathbf{L}$, a vector whose intensity is $\sqrt{l(l+1)}\,\hbar$ and a direction that does not tell us much. An electron in orbit, like a circular current, produces a magnetic field that can interact with external magnetic fields. Therefore, the external field gives us confirmation of the direction and direction of the orbital angular momentum vector in the hydrogen atom. By convention, we set the z axis along the direction of the magnetic field $\mathbf{B}$. Then m_l gives the component $\mathbf{L}$ in the direction $\mathbf{B}$

$$L_z = m_l \hbar, \tag{2.194}$$

with m_l being at most equal to l, so L_z is always less than L. It's a seemingly "weird" thing in quantum mechanics, which also makes sense.



Slika 2.16: Magnetic moment.

In the picture 2.16 we see that if L shows "exactly" the direction of z-axe (magnetic) then the electron orbit lies "exactly" in the Oxy plane, so the indefiniteness of the moment

in in the z-axe direction is "infinite". As this is impossible, for an electron in an atom, the deflections of z-axe are constantly present. This is why L_z is generally less than L.

After breaking down the wave function into factors (2.180) by separating the variables and the special solutions of the Schrödinger equation in spherical coordinates, we define *probability density* with

$$\Pr(r,\theta,\phi) = \psi^*\psi dV = \Pr(r)\Pr(\theta)\Pr(\phi)dV, \tag{2.195}$$

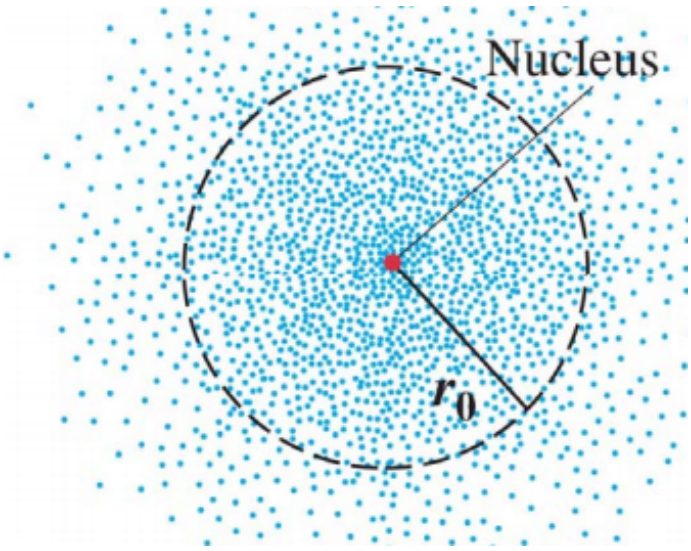
where $\Pr(\omega)$ is the probability of outcome ω and dV is the infinitesimal element of the volume in spherical coordinates

$$dV = r^2 \sin\theta\, dr d\theta d\phi. \tag{2.196}$$

Because the variables are separated and the functions R, Θ, and Φ are orthonormal, the integral of the probability density can be viewed as a triple integral, that is, the product of three one-dimensional integrals. The product of these integrals is also normed on unit, and the special integrals are:

$$\begin{cases} \Pr(r) = \int R^*(r)R(r)r^2\, dR, \\ \Pr(\theta) = \int \Theta^*(\theta)\Theta(\theta)\sin\theta\, d\theta, \\ \Pr(\phi) = \int \Phi^*(\phi)\Phi(\phi)\, d\phi. \end{cases} \tag{2.197}$$

Maximum probability occurs at the peaks of the probability nucleus, where the probability densities are greatest.



Slika 2.17: Густина вероватноће електрона.

The average value of, say r_0, is equal to the expected value $\bar{r}$. The most probable value of r for 1s electron is $r_0 = a_0$, which it should be, but the average value of r is $\bar{r} = 1.5a_0$, which is fine in quantum theory but not in the earlier model.

Quantum mechanics has made other corrections to the previous view of the atom. For example, we only have probabilities of finding electrons using a given coordinate system. An electron does not move around the Nucleus of an atom in any conventional sense. The fuzzy cloud in the 2.17 image does not show electrons in the nucleus of atoms, but the densities

of the probability of finding a single electron of the first orbit. The denser a cloud is in an image spot, the more likely it is to find electrons in that spot.

The representation of Hilbert algebra is a quantum system, its vectors are quantum states, and linear unitary operators are quantum processes, evolutions of quantum states. In the end, quantum states are always some particles simply because "every property of information is discrete". But as the aforementioned vector space operators themselves form a new vector space, *dual space* with the first, so are the quantum processes some "particles" too which are again subject to the same quantum laws.

To make things even more complicated, the exponential and logarithmic functions of vectors are also vectors. This confuses us when we move from probabilities, wave functions of quantum states in the usual sense, to logarithms, then *information of perception*, or vice versa, when we pass from information to their exponential functions that become again probabilities, that is, the "classical" quantum states.

Afterword

It is shown why the 'information of perception' is some information and where its extremes are, then that it can be decomposed into a much dimensional space of observable (vectors), and then reduced to only one plane 'observable'. Its invariance is followed by Lorentz transformations, also 2-dim as well as a free point-to-point drop in the gravitational field. The principle of minimalism of information in all these and similar considerations can be replaced by the principle of the least effect, and vice versa, so from the information of perception Riemann and gravitational geodesic lines can be proved, as Einstein's general equations can be derived. Perception information agrees with the Schrödinger equation. The rest is an elaboration of this idea.

Rastko Vuković, April 2019.

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